Optimizing the Inclination Angle of a Solar PV System for Installation in Slave Lake, Alberta, for Maximizing PV Power Generation

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Abstract: Increasing fossil fuel prices, electricity demand, and global concern for greenhouse gas emissions have generated an increase in research of novel renewable energy technologies for green power generation. Renewable solar-photovoltaic (PV) systems are known fosr their ability to directly convert the solar energy into electrical energy for locations where there is a desperate need and demand for electrical power, e.g. Communities and towns in remote locations, and solar energy is available. PV systems tend to generate more power if they are operated efficiently. The inclination (tilt) angle at which a solar photovoltaic (PV) module is sloped from the horizontal plane is one of the most influencing system parameters that affects the electrical power output from the PV system. This is due to the fact that the variation in the tilt angle affects the amount of incident solar radiation received on a PV system and utilized by the load once installed at a certain site. In this work, a mathematical model is used to estimate the total solar radiation incident on an inclined surface and to determine the optimum inclination angle for maximizing power generation from a solar PV system installed in Slave Lake, Alberta, Canada. The total solar energy received on the optimally inclined PV surface is computed for all months in a year. It was found that the monthly average optimum tilt angle of the PV system varied from a minimum value of 9^o in the month of June to a maximum value of 78^o in the months of January and December. The yearly average monthly optimum tilt angle for the fixed PV system in Slave Lake was estimated to be at 47^o. It was also found that the highest maximum incident radiation of approximately 21.30 Mj/m^2 for the whole year occurred in the month of June, where as approximately 30.4% of this value (the lowest in the year) occurred in the month of December. The minimum incident solar radiation for all months in the winter season occurred at almost zero tilt angle and that the variation of the monthly average daily sunlight hours varied from 6.9 hours in December to as high as 17.1 hours in June.

Keywords: Clean renewable energy, power generation, solar PV systems, Slave Lake, Alberta, maximum power generation, PV inclination angle, tilt angle, solar energy, Canada.

1. INTRODUCTION

Slave Lake (latitude = 55.3^o N, longitude =114.8^o W) is a small remote town located approximately 248 km northwest of Edmonton, Alberta, Canada. It hosted the Arctic Winter Games in 1994 [1]. The town has a land area of approximately 14.18 km^2 located on the southeast shore of Lesser Slave Lake with a population of approximately 6,782 (Canada Census 2011) [2]. In Slave Lake, the maximum and minimum average monthly temperatures of 18 °C and -18 °C occur in the months of July and January, respectively [3]. In Canada, Solar photovoltaic (PV) technology has become a favored form of renewable energy technology for direct electrical power generation due to a number of social and economic factors, including the need to reduce greenhouse gas (GHG) emissions, deregulation, and the restructuring of electric power generating companies. The rapid growth in the installation of solar PV systems recently indicates that the technology is rapidly gaining grounds in Canada [4]. Slave Lake has annual averaged daily sunshine hours of approximately 12.3 hours [5,6], and a monthly average daily insolation incident on a horizontal surface of approximately 11.592 MJ/m^2 [6].

The inclination (tilt) angle at which a solar photovoltaic (PV) module is sloped from the horizontal plane, is one of the most influencing system parameters that affect the electrical power output from a solar PV system. This is due to the fact that the variation in the tilt angle affects the amount of incident solar radiation received on a PV system and utilized by the load once installed at a certain site [7,8]. Therefore, in order to maximize the overall power production of a PV system throughout a year, the tilt angle of the PV system should be optimized for a given geographical site, considering PV systems fixed at a certain angle throughout the year in that site. In addition, positioning the PV system at an optimum tilt angle leads to lesser PV array area required to match a fixed electrical load, thus reducing the capital cost of the PV system. This also applies to thermal solar collectors used for heating applications in general. A number of studies were conducted for optimization of tilt angles for different applications using various methods [9-13]. The utilization of solar PV systems for electrical power generation is a useful and convenient technology in

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E-ISSN: 2409-5818/15

Slave Lake due to its relatively remote location, the increasing demand and community consumption for electrical power, the lack of other sources of electricity, and the abundance of renewable solar energy. Therefore, it would be more efficient to maximize the performance of PV systems installed in Slave Lake by optimizing the inclination angle of these systems in order to maximize the utilization of the incident solar energy on the PV systems and their power production and justify their economics. In this paper, a widely used mathematical model, developed by Liu and Jordan (1962) [14] and extended by Klein (1977) [15], is used to estimate the total solar radiation incident on an inclined PV surface and to determine the optimum inclination angle for maximizing power generation from a solar PV system installed in Slave Lake, Alberta, Canada. In this work, the total solar energy received on the optimally inclined PV surface is computed for all months in a year and presented for the four seasons in a year; namely, winter, spring, summer, and autumn. Also, the yearly averaged optimum tilt angle for the PV system is determined.

2. MATHEMATICAL MODEL AND COMPUTATION METHODOLOGY

The monthly average daily solar radiation, $\bar{H}_T$, impinging on an unshaded inclined (tilted) surface with an angle β from a horizontal plane on a site at a latitude φ north of Equator and directly pointed towards the Equator (so that the solar azimuth angle $\gamma = 0$) can be estimated using the widely used mathematical model developed by [9,10] given by:

$$\bar{H}_T = \bar{H}_{b,T} + \bar{H}_{d,T} + \bar{H}_{g,T} \tag{1}$$

The PV system orientation using $\gamma = 0$ for any location in northern hemisphere (north of Equator), such as Slave Lake location, is considered to be the optimum angle for receiving an overall maximum solar radiation over the year [7]. In Equation (1), $\bar{H}_T$ (MJ/m^2) is modeled as the sum of three solar radiation components: (1) the beam (direct) radiation incident on the tilted surface, $\bar{H}_{b,T}$ (2) the sky-diffuse radiation received on the tilted surface $\bar{H}_{d,T}$, and (3) the ground-reflected radiation received on the tilted surface $\bar{H}_{g,T}$. Mathematically, these three components of solar radiation are expressed using the following equations:

$$\bar{H}_{b,T} = \bar{H}\left(1 - \frac{\bar{H}_d}{\bar{H}}\right)\bar{R}_b \tag{2}$$

$$\bar{H}_{d,T} = \bar{H}_d\left(\frac{1+\cos\beta}{2}\right) \tag{3}$$

$$\bar{H}_{g,T} = \bar{H}\,\bar{\rho}_g\left(\frac{1-\cos\beta}{2}\right) \tag{4}$$

It should be noted that the sky-diffuse and ground-reflected components of radiation on the tilted surface are each assumed to be isotropic (i.e. this model is referred to as the isotropic sky model). **Figure 1** shows the basic solar angles used in the above model [16]. In the above equations, the term $\bar{H}$ (MJ/m^2) is the monthly average daily total terrestrial radiation incident on a horizontal surface (i.e. $\beta = 0$); $\bar{H}_d$ (MJ/m^2) is the monthly average daily sky-diffuse radiation incident on a horizontal surface; and $\bar{\rho}_g$ is the monthly average daily ground reflectivity (also known as albedo). The ratio $\frac{\bar{H}_d}{\bar{H}}$ is computed using the following correlations [9,10]:

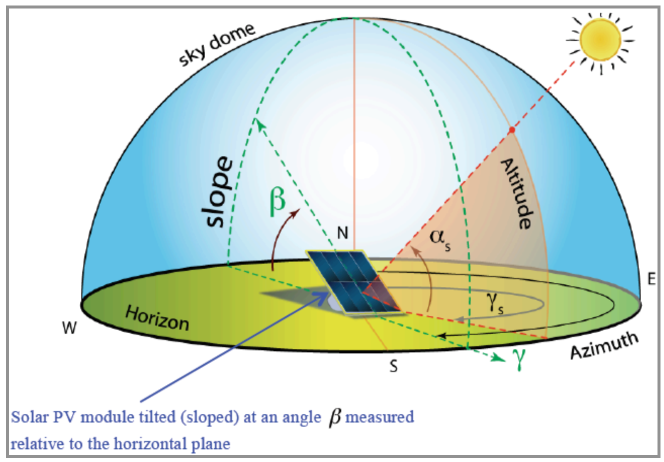

Figure 1: A conceptual illustration showing basic solar angles used in this study [16].

(a) For $\omega_s \le 81.4^o$ and $0.3 \le \bar{K}_T \le 0.8$, the following correlation is used:

$$\left(\frac{\bar{H}_d}{\bar{H}}\right) = 1.391 - 3.560\,\bar{K}_T + 4.189\,(\bar{K}_T)^2 - 2.137\,(\bar{K}_T)^3 \tag{5}$$

(b) For $\omega_s > 81.4^o$ and $0.3 \le \bar{K}_T \le 0.8$, the following correlation is used:

$$\left(\frac{\bar{H}_d}{\bar{H}}\right) = 1.311 - 3.022\,\bar{K}_T + 3.427\,(\bar{K}_T)^2 - 1.821\,(\bar{K}_T)^3 \tag{6}$$

In both correlations above, the sunset hour angle ω_s is given by:

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \tag{7}$$

Where, the solar declination angle δ as a function of the day in the year *n* is computed using:

$$\delta = 23.45 \sin\left[360 \ \frac{(284+n)}{365}\right] \tag{8}$$

The monthly average daily clearness index $\bar{K}_T$ appearing in Equations (5) and (6) above is defined as the ratio of the terrestrial radiation to the extraterrestrial radiation received on a horizontal surface. It is a measure of the effect of the atmosphere conditions. It is estimated using:

$$\bar{K}_T = \frac{\bar{H}}{\bar{H}_o} \tag{9}$$

Where, $\bar{H}_o$(MJ/m^2) is monthly average daily extraterrestrial radiation received on a horizontal surface, estimated by:

$$\bar{H}_o = \frac{24*3600*1367}{\pi}\left(1+0.033\cos\frac{360*n}{365}\right) *\left(\cos\phi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\phi\sin\delta\right) \tag{10}$$

As indicated in Equation (10), the extraterrestrial radiation here is a function of the solar angles and mean day in the month. In this work, $\bar{H}$ values for Slave Lake location (based on monthly averaged values over 22 years of data [17]) and $\bar{H}_o$ are compared and shown in Figure **2**. In Equation (2), the parameter $\bar{R}_b$ is known as the beam geometric factor for the mean day of the month, expressed by:

$$\bar{R}_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_s' + (\pi/180)\ \omega_s'\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + (\pi/180)\ \omega_s\sin\phi\sin\delta} \tag{11}$$

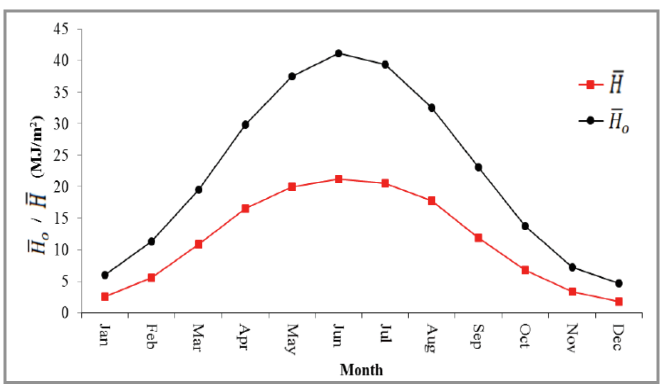

Figure 2: Comparison of the monthly average daily solar radiation (extraterrestrial vs. terrestrial) incident on a horizontal surface (PV module) in Slave Lake, Alberta.

Where, ω_s' is the sunset hour angle for the tilted surface for the mean day of the month, which is given by:

$$\omega_s' = \min\begin{bmatrix}\cos^{-1}(-\tan\phi\tan\delta) \\ \cos^{-1}(-\tan(\phi-\beta)\tan\delta)\end{bmatrix} \tag{12}$$

Where, “min” means the smaller of the two terms in the brackets. The monthly average daily number of sunlight hours can be computed using:

$$N = \frac{2}{15}\cos^{-1}(-\tan\phi\tan\delta) \tag{13}$$

The yearly average monthly optimum tilt angle for the PV system installed in Slave Lake can be determined using the simple relationship given by:

$$\left(\beta_{opt}\right)_{year} = \frac{\sum_{Jan}^{Dec}\beta_{opt}}{12} \tag{14}$$

In this study, the computation procedure was implemented systematically using the set of Equations (1) to (12) and by varying the inclination (tilt) angle β from 0^0 to 90^o with an increment of 3^o for every month in the year considering the mean day for every month. The results of $\bar{H}_T$ pertaining to all set of angles for a particular month were compared and the tilt angle that produces the maximum value for $\bar{H}_T$ (thus, called $\bar{H}_{T,\max}$) was assigned as the optimum tilt angle for that month in the year. The computation procedure was repeated for all months in the year and the optimum inclination angles for all months for the PV system installed in Slave Lake were determined.

3. RESULTS AND DISCUSSION

Following the mathematical model and computation procedure presented in the previous section, the results are presented and discussed in this section. Figure **3** shows a comparison for the monthly average daily clearness index for all months in the year. The maximum clearness index occurred in the month of

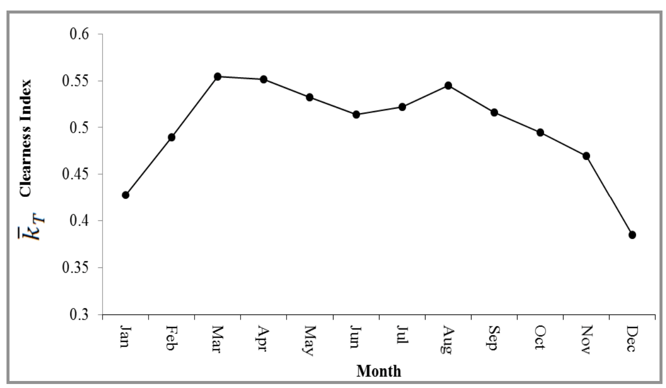

Figure 3: The estimated monthly average daily clearness index for a solar PV module in Slave Lake, Alberta.

March with a value of approximately 55%, whereas the minimum value of approximately 38% occurred in December. The ratio of sky-diffuse to total radiation on a horizontal PV system was determined for all months in the year and compared in Figure **4**. The ratio varied from approximately 0.37 to 0.53 with maximum value being in the month of December and minimum being in the month of March. This indicates that the value of the monthly average daily sky-diffuse component of radiation received on a horizontal surface in Slave Lake, Alberta, was found maximum in the month of December, which is nearly 53% of its corresponding $\bar{H}$ value. Comparison of the estimated monthly average daily solar radiation incident on a tilted PV system in Slave Lake, Alberta, for the months of December, January and February (representing the cold winter season) as a function of tilt angle, is shown in Figure **5**. The highest estimated values of $\bar{H}_T$ occurred in the month of February for all tilt angles compared to the months of December and January. In particular, at a tilt angle of 69°, the maximum incident solar radiation in the month of February was approximately 12.75 MJ/m^2. However, the maximum incident solar radiation for the months of December and January occurred both at the same tilt angle of approximately 78° with values of 6.47 MJ/m^2 and 8.45 MJ/m^2, respectively. The minimum incident solar radiation for all months in the winter season occurred at a tilt angle = 0 (i.e. the PV system is positioned horizontally). It should be noted that the incident solar radiation initially increased rapidly and monotonically for the three months as the tilt angle increased from horizontal position until the tilt angle is approximately 50° after which the increase in the incident solar radiation became relatively smaller. For this case, the average optimum tilt angle that produced average maximum incident solar radiation on the PV system installed in Slave Lake for the winter season was approximately 75°.

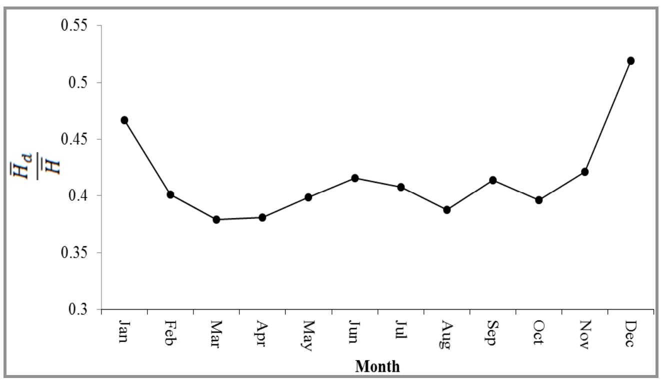

Figure 4: The estimated ratio of monthly average daily diffuse solar radiation to global radiation incident on a horizontal PV module in Slave Lake, Alberta.

The estimated monthly average daily solar radiation incident on a tilted PV system in Slave Lake for the months of March, April and May (representing the spring season) as a function of tilt angle, is compared and shown in Figure **6**. For these three months, the maximum incident solar radiation occurred at significantly different tilt angles. For example, the maximum incident solar radiation of approximately 16.76 MJ/m^2 for the month of March in Slave Lake occurred at an optimum tilt angle of 57°. However, for the months of April and May, the maximum incident solar radiation of fairly higher values approximately 19.09 MJ/m^2 and 20.59 MJ/m^2 occurred at relatively smaller optimum tilt angles of 36° and 18°, respectively. It is interesting to note that for the month of May, the incident solar radiation initially increased slowly and gradually as the tilt angle increased from 0 to 18° and

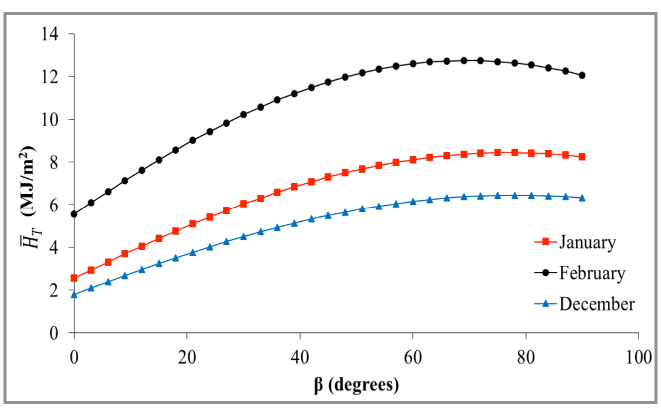

Figure 5: Comparison of the estimated monthly average daily solar radiation incident on a tilted PV module in Slave Lake, Alberta, for the cold season (months of December, January and February) as a function of tilt angle.

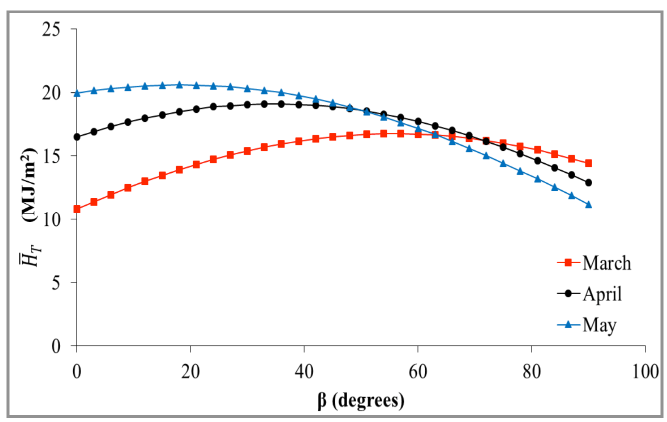

Figure 6: Comparison of the estimated monthly average daily solar radiation incident on a tilted PV module in Slave Lake, Alberta, for the spring season (months of March, April and May) as a function of tilt angle.

then started to decrease rapidly as the tilt angle continued to increase until the incident radiation hits a minimum value of approximately 11.20 MJ/m^2 as the PV system is positioned vertically (i.e. tilt angle = 90^o). For the spring season in Slave Lake, the highest minimum value of the monthly average incident solar radiation of approximately 14.43 MJ/m^2 on the vertically positioned PV system occurred in the month of March. It is also interesting to note that at a tilt angle of approximately 66^o, the value of the incident solar radiation is almost the same for the three months being approximately 16.17 MJ/m^2. The average optimum tilt angle of the PV system installed in Slave Lake for the spring season was approximately 37^o. Figure **7** shows a comparison of the estimated monthly average daily incident solar radiation in Slave Lake for the summer season (months of June, July and August) as a function of tilt angle of the PV module system. For this case, the incident solar radiation profiles look very similar for the months of June and July whereas they differ slightly from the month of August. The optimum tilt angle of the PV system was the lowest being 9^o in the month of June with a maximum incident solar radiation of nearly 21.30 MJ/m^2 that slightly dropped to 20.87 MJ/m^2 in the month of July at an optimum tilt angle of 15^o. The optimum tilt angle was relatively higher being 27^o in the month of August with lesser value of the maximum incident solar radiation of approximately 19.32 MJ/m^2 relative to the months of June and July. The average optimum angle of the PV system for the summer season in Slave Lake was computed to be approximately 17^o with an average maximum incident solar radiation of nearly 20.50 MJ/m^2.

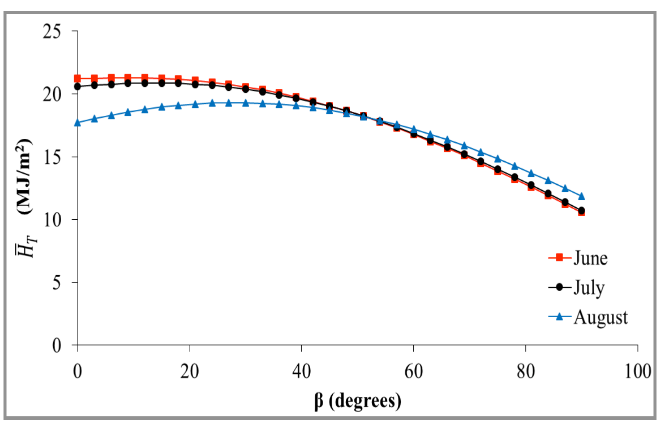

Figure 7: Comparison of the estimated monthly average daily solar radiation incident on a tilted PV module in Slave Lake, Alberta, for the summer season (months of June, July and August) as a function of tilt angle.

The estimated monthly average daily solar radiation incident on the inclined PV system in Slave Lake for the months of September, October and November (representing the Autumn season) as a function of tilt angle, is compared and shown in Figure **8**. For these three months, the variation of the incident solar radiation with respect to the tilt angle of the PV system looks more considerable. The results show that the highest incident solar radiation occurred for the month of September for all tilt angles with an exception of 90^o at which the value of the incident radiation for this month of approximately 12.12 MJ/m^2 matches that for the month of October. The optimum tilt angles for the months of September, October and November are 45^o, 63^o, and 75^o, respectively, and their respective estimated amounts of maximum incident solar radiation are 15.62, 13.27, and 10.39 MJ/m^2. It is interesting to note that the increase in the incident solar radiation was rapid at smaller tilt angles for all the months in the season. The average optimum tilt angle for autumn season is approximately 61^o. Figure **9** shows the comparison of the estimated values of maximum

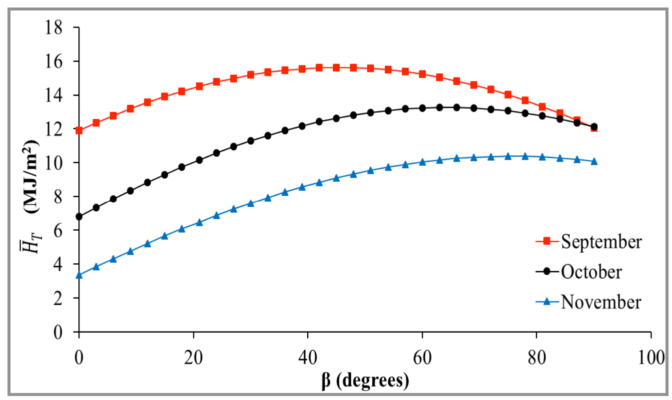

Figure 8: Comparison of the estimated monthly average daily solar radiation incident on a tilted PV module in Slave Lake, Alberta, for the autumn season (months of September, October and November) as a function of tilt angle.

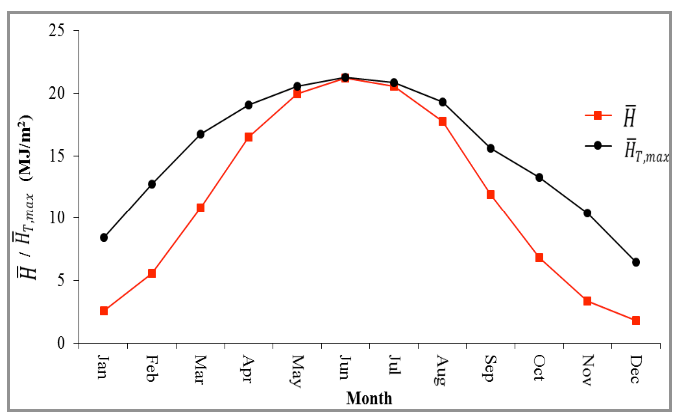

Figure 9: Comparison of the computed maximum monthly average daily solar radiation incident on an optimally tilted PV module in Slave Lake, Alberta, as a function of the month in the year.

monthly average daily solar radiation incident on an optimally tilted PV module in Slave Lake, Alberta, as a function of the month in the year. The highest maximum incident radiation of approximately 21.30 MJ/m^2 for the whole year occurred in the month of June, whereas approximately 30.4% of this value (the lowest in the year) occurred in the month of December. It is worth noting that the optimum tilt angle for the month of June is nearly horizontal with only 9^0. Table **1** summarizes the results of the monthly optimum tilt angles and their respective maximum incident monthly average daily solar radiation for the tilted PV system installed in Slave Lake, Alberta. The yearly average monthly optimum tilt angle for the PV system in Slave Lake was estimated to be 47^o. The variation of the monthly average daily sunlight hours varied from 6.9 hours in December to as high as 17.1 hours in June.

Table 1: Summary of Monthly Optimum Tilt Angles and Their Corresponding Maximum Incident Monthly Average Daily Solar Radiation for the Tilted PV System Installed in Slave Lake, Alberta

Month	β_{opt} (Degrees)	$\bar{H}_{T,\max}$ (MJ/m^2)	N (Hours)
Jan	78	8.45	7.5
Feb	69	12.75	9.4
Mar	57	16.76	11.5
Apr	36	19.09	13.8
May	18	20.59	15.9
Jun	9	21.30	17.1
Jul	15	20.87	16.5
Aug	27	19.32	14.7
Sep	45	15.62	12.4
Oct	63	13.27	10.1
Nov	75	10.39	8.1
Dec	78	6.47	6.9
	$\left(\bar{\beta}_{opt}\right)_{year}$ = 47°		

4. CONCLUSIONS

In Canada, Solar photovoltaic (PV) technology has become a favored form of renewable energy technology for direct electrical power generation due to a number of social and economic factors, including the need to reduce greenhouse gas (GHG) emissions, deregulation, and the restructuring of electric power generating companies. The rapid growth in the installation of solar PV systems recently indicates that the technology is rapidly gaining grounds in Canada. Slave Lake is a small remote town located approximately 248 km northwest of Edmonton, Alberta, Canada. The utilization of solar PV systems for electrical power generation is a useful and convenient technology in Slave Lake due to its relatively remote location, the increasing demand and community consumption for electrical power, the lack of other sources of electricity, and the abundance of renewable solar energy. PV systems tend to generate more power if they are operated efficiently and consistently. The inclination (tilt) angle at which a solar PV module is sloped from the horizontal plane is one of the most influencing system parameters that affects the electrical power output from the PV system. This is due to the fact that the variation in the tilt angle affects the amount of incident solar radiation received on a PV system and utilized by the load once installed at a certain site. In this work, a mathematical model was used to estimate the total solar radiation incident on an inclined surface and to determine the optimum inclination angle for maximizing power generation from a solar PV system installed in Slave Lake, Alberta, Canada. It was found that the monthly average optimum tilt angle of the PV system varied from a minimum value of 9^o in the month of June to a maximum value of 78^o in the months of January and December. The yearly average monthly optimum tilt angle for the fixed PV system in Slave Lake was estimated to be at 47^o. It was also found that the highest maximum incident radiation of approximately 21.30 MJ/m^2 for the whole year occurred in the month of June, whereas approximately 30.4% of this value (the lowest in the year) occurred in the month of December.

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Received on 01-06-2015 Accepted on 30-05-2015 Published on 31-12-2015

DOI: http://dx.doi.org/10.15377/2409-5818.2015.02.02.3