Mass Transfer Resistances at the Boundary of a Fractured Porous Medium

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Abstract: The aim of this paper is the study of the mass transfer resistance effects at the boundary of a fractured porous media. The boundary between the porous media adjacent to the fluid considers the transient effects. The numerical experiments show that the α parameter has an influence that facilitates the mass transfer of the porous region to the fluid region. The α parameter expresses the relation of the mass transfer resistances between the porous media and the homogeneous fluid; in the present work it is considered as a parameter which facilities mass transfer of the porous region to the fluid region.

Keywords: Averaging volume, fractured porous media, surface transport equation, mass transfer, numerical model, interfacial effects.

1. INTRODUCTION

Due to the transport properties of fractures, the flow through a fractured porous media differs drastically from that in a porous medium formed by intergranular porosity. The porous matrix provides the main storage for the fluids and the transport takes place in the fracture, whose study is of practical importance, e.g. the fractured petroleum reservoirs represent over 20% of the world´s oil reserves [1]. Fractured reservoirs are also found in the geothermal industry, where water is reinjected to reservoirs to maintain reservoir pressure [2].

The interfacial region between porous and fluid regions is essential for the process of mass transport, energy and momentum from the porous region to the fluid region, and for many years it has been the subject of numerous studies. The boundary region between a fluid saturated porous medium and an adjacent horizontal fluid layer was studied by Beavers and Joseph [3]. These authors presented a semi-empirical jump boundary condition to describe the process at the inter-region. Neale and Nader [4] analyzed a similar system and they introduced the Brinkman term for the porous side. In the decade of the 1980s, others studies emerged, e.g. [5-8]. It was not until the 1990's when these kind of studies were intensified with the works, e.g. [9-13]. However, in the new century it is important to highlight the work of Alazmi and Vafai [14], Espinosa-Paredes [15], and Espinosa-Paredes *et al*. [16].

In this work a numerical analysis for mass transfer resistances in transient regime of the process of mass transport at the boundary between a porous media and a fluid layer is studied in detail. In this study the porous media fractured considered has three regions: (1) heterogeneous medium, (2) homogeneous medium, and (3) boundary region between the heterogeneous and homogeneous mediums. A previous work of Gwo *et al*. [17] considers the mass transfer with a two-region model. In this work the model of Espinosa-Paredes *et al*. [16] is applied for the mass transport process in the interfacial region.

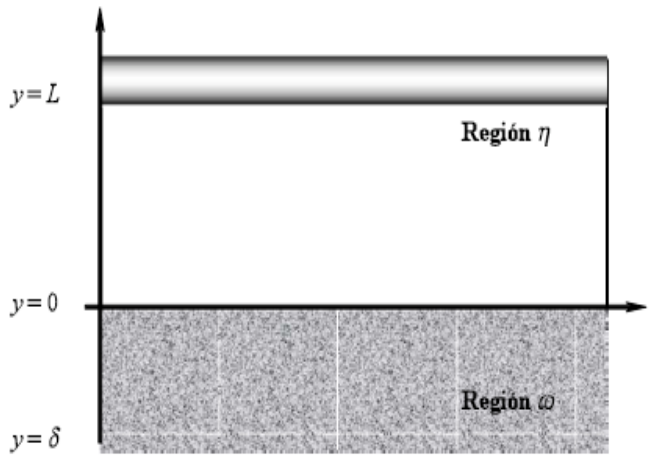

Figure 1: Schematic representation of the one-dimensional mass transfer two-regions and interfacial region. Where ω-region represents the porous medium, η-region is the fluid region, and interfacial region is located in $y = 0$.

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2. MATHEMATICAL MODEL

The system under study is illustrated in Figure **1**, where the porous media is assumed as isotropic and homogeneous, with periodic structure and two impermeable boundaries at $y = L$, and $y = -\delta$. The interfacial region between the porous and fluid regions is located at $y = 0$. Therefore, the three-region model that describes the mass transfer process in the system illustrated by Figure **1**, is given by Espinosa-Paredes *et al*. [16],

Fluid region (η),

$$\frac{\partial \langle C_{A\beta} \rangle_\eta^\beta}{\partial t} + \langle v_{\beta x} \rangle_\eta^\beta \frac{\partial \langle C_{A\beta} \rangle_\eta^\beta}{\partial y} = D_\eta \frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}\langle C_{A\beta} \rangle_\eta^\beta\right], y \in [L,0] \quad (1)$$

Porous region (ω),

$$\left(\varepsilon_\beta + a_{\sigma\beta} K_{eq}\right)_\omega \frac{\partial \langle C_{A\beta} \rangle_\omega^\beta}{\partial t} = \frac{\partial}{\partial y}\left[\varepsilon_{\beta\omega} D_\omega \frac{\partial \langle C_{A\beta} \rangle_\omega^\beta}{\partial y}\right], y \in [0,\delta] \quad (2)$$

Boundary region ($\omega - \eta$),

$$\varepsilon_{\beta\omega} D_\omega \frac{\partial \langle C_{A\beta} \rangle_\omega^\beta}{\partial y} - D_\eta \frac{\partial \langle C_{A\beta} \rangle_\eta^\beta}{\partial y} = \gamma a_{\sigma\beta} K_{eq} \frac{\partial \langle C_{A\beta} \rangle_\omega^\beta}{\partial t}, at \quad y = 0 \quad (3)$$

Initial and boundary conditions,

I.C.1 $$\langle C_{A\beta} \rangle_\eta^\beta = \langle C_{A\beta} \rangle_{\eta,0,}^\beta \quad for \quad t = 0 \quad (4)$$

I.C.2 $$\langle C_{A\beta} \rangle_\omega^\beta = \langle C_{A\beta} \rangle_{\omega,0,}^\beta \quad for \quad t = 0 \quad (5)$$

B.C.1 $$\frac{\partial \langle C_{A\beta} \rangle_\eta^\beta}{\partial y} = 0, \quad at \quad y = L \quad (6)$$

B.C.2. $$\langle C_{A\beta} \rangle_\omega^\beta = \langle C_{A\beta} \rangle_\omega^\beta, \quad at \quad y = 0 \quad (7)$$

B.C. 3 $$\frac{\partial \langle C_{A\beta} \rangle_\omega^\beta}{\partial y} = 0, \quad at \quad y = \delta \quad (8)$$

The form of the inter-region boundary condition given by Eq. (3) is consistent with the literature dealing with jump condition at the phase interfaces [18-19]. The transport equations, Eqs. (1) and (2), are coupled by the interfacial boundary condition given by Eq. (3), which includes mass transfer resistance and accumulation effects.

3.1. Dimensionless Equations

For convenience of mathematical formulation, we introduce the following dimensionless variables,

$$U_f = \frac{\langle C_{A\beta} \rangle_\eta^\beta}{C_\eta}, \quad U_p = \frac{\langle C_{A\beta} \rangle_\omega^\beta}{C_\eta}, \quad \tau = t\frac{D_\eta}{L^2}, \quad Y = \frac{y}{L} \quad (9)$$

Then, the dimensionless form of the problem defined by Eqs. (1)-(8), is now defined by,

$$\frac{\partial U_f}{\partial \tau} + Pe\frac{\partial U_f}{\partial Y} = \frac{\partial U_f}{\partial Y^2}, \quad Y \in [1,0], \qquad \textit{Fluid region } (\eta) \quad (10)$$

$$\frac{\partial U_p}{\partial \tau} = \alpha \frac{\partial^2 U_p}{\partial Y^2}, \quad Y \in [0,-\delta/L], \qquad \textit{Porous region } (\omega) \quad (11)$$

$$\lambda \frac{\partial U_p}{\partial Y} - \frac{\partial U_f}{\partial Y} = \varphi \frac{\partial U_p}{\partial \tau}, \quad atY = 0 \qquad \textit{Boundary region } (\omega - \eta) \quad (12)$$

Initial and boundary conditions,

I.C.1* $$U_f = U_{f0}, 0 \le Y \le 1, \quad for\ \tau = 0 \quad (13)$$

I.C.2* $$U_p = U_{p0}, 0 \le Y \le -\delta/L, \quad for\ \tau = 0 \quad (14)$$

B.C.1* $$\frac{\partial U_f}{\partial Y} = 0, \quad at \quad Y = 1 \quad (15)$$

B.C.2* $$U_f = U_p, \quad at \quad Y = 0 \quad (16)$$

B.C.3* $$\frac{\partial U_p}{\partial Y} = 0, \quad at \quad Y = -\frac{\delta}{L} \quad (17)$$

where

$$Pe = \frac{\left\langle v_{\beta x} \right\rangle_{\eta}^{\beta} L}{D_{\eta}}, \lambda = \frac{\varepsilon_{\beta\omega} D_{\omega}}{D_{\eta}} \quad , \qquad \varphi = \frac{\gamma a_{\sigma\beta} K_{eq}}{L} \tag{18}$$

One of the advantages of homogenization is that it generates different up-scaled models when characteristic dimensionless numbers assume values of different orders of magnitude with respect to $\varepsilon_{\beta\omega}$. Other dimensionless numbers such as α, λ, and φ follow from the up-scaled model equations.

4. NUMERICAL SOLUTION

In the present solution, the fractured porous media is represented by a one-dimensional, mesh-centered grid consisting of a variable number of vertical elements. The governing equations are written for each element of the grid. The differential equations described previously are transformed into discrete equations using the technique of finite differences in explicit form. The spatial discretization *scheme* used in the present case is known as central-differences while forward-differences are used for the time derivative. Application of these discrete forms enables the equations for each region to be written as a single set of equations for the sweep in the vertical direction.

4.1. Resistances between Porous Media and Homogeneous Fluid

In order to study the process of mass transport from the porous region to the fluid region, the initial conditions are: $U_{p0} = 1$, and $U_{f0} = 0$

In this work $\lambda = \frac{2\varepsilon_{\beta\omega}^2}{3 - \varepsilon_{\beta\omega}}$ obtained with the model for isotropic media [20], which is used to calculate the parameter α:

$$\alpha = \frac{D_{\omega}}{\left(1 + \frac{a_{\sigma\beta\omega} K_{eq}}{\varepsilon_{\beta\omega}}\right) D_{\eta}} \tag{19}$$

The α parameter expresses the relation of the mass transfer resistances between the porous media and the homogeneous fluid. Then, in order to study the effect of the mass transfer by diffusion under transient conditions, we present the results of numerical experiments in Figures **2-4**, for four values of α: 0.1, 0.5, 0.9, and 1.0. Figures **2** and **3** show the behavior of the average concentration in the fluid region, $\langle U_f \rangle$, and average concentration in the porous region $\langle U_p \rangle$, which are given by,

$$\langle U_f \rangle = \int_0^1 U_f \, dY \tag{20}$$

and

$$\langle U_p \rangle = \frac{L}{\delta} \int_0^{-\frac{\delta}{L}} U_p \, dY, \tag{21}$$

respectively.

The concentration in the interfacial region, $U_f \big|_{Y=0}$, is shown in Figure **4**. The discussion of the behavior of the numerical results in this work is performed using the average concentrations for fluid and porous regions.

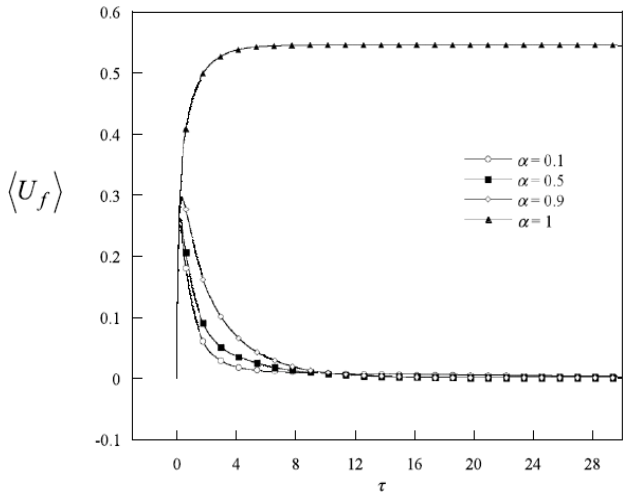

Figure 2: Influence of the parameter α (Eq. **19**) on the average concentration of the fluid region, with $Pe = 1$ and $\varphi = 1$.

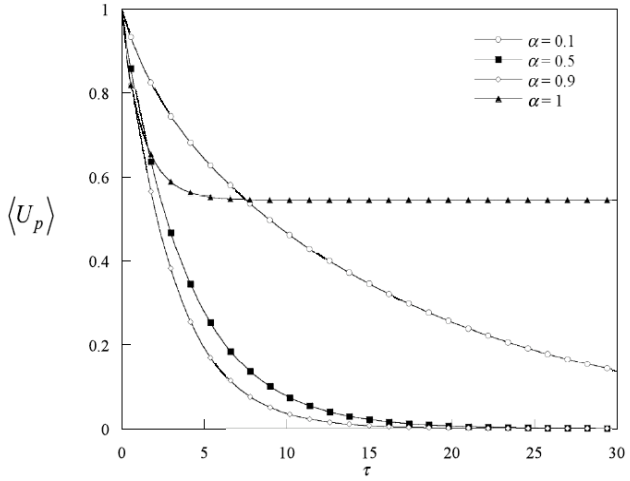

Figure 3: Influence of the parameter α (Eq. **19**) on the average concentration of the porous region, with $Pe = 1$ and $\varphi = 1$.

The influence of the parameter α on the $\langle U_f \rangle$ is presented in Figure **2**. As it can be observed in this Figure, for $\alpha < 1$, the concentration reaches a maximum and then decays exponentially with time, which implies that the mass transfer resistance in the porous region is greater than the mass transfer resistance in the fluid region, as expected. If $\alpha = 1$, the mass transfer resistance in both regions is equivalent, which represents the limit case where the entire system is homogeneous. In this case, it can be observed that the steady state is reached before $\alpha < 1$. In some applications where the mass transfer process is crucial, as for example in oil recovery in naturally fractured systems, the effects of mass transfer resistance is the technological challenge.

Figure **3** depicted the transient behavior of the average concentration in the porous region, which decays exponentially with dimensionless time for $\alpha < 1$. In general for $\alpha < 1$ the concentration in the porous region tends to zero, and this decay is slower as α decreases. When $\alpha = 1$, the concentration reaches steady state at a value different from zero.

The physical meaning of this behavior is due to mass transfer resistance. In all cases analyzed for $\alpha < 1$, the resistance to the mass transfer in the porous region is greater than in the fluid region, i.e., $D_\omega < D_\eta^*$, where $D_\eta^* = \left(1 + a_{\sigma\beta\omega} K_{eq} / \varepsilon_{\beta\omega}\right) D_\eta$. When $\alpha = 1$, the resistances to the mass transfer in both regions are equivalent, i.e., $D_\omega = D_\eta^*$ and the system reaches the local mass equilibrium in a nonzero value, where the differences of the concentrations between regions maintain a mass diffusion process.

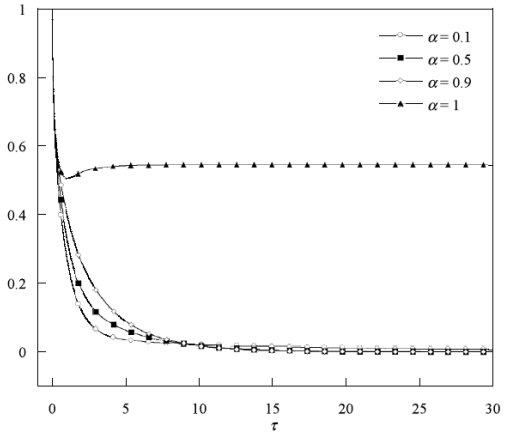

Figure 4: Influence of the parameter α (Eq. **19**) on the interfacial concentration, with $Pe = 1$ and $\varphi = 1$.

The interfacial behavior is shown in Figure **4**. As it can be observed, the interfacial concentration exhibits practically the same behavior that the concentration in the fluid and porous region (Figures **2** and **3**, respectively). However, it is important to note that the interfacial phenomena govern the behavior in the adjacent regions, due that the interface region couples the porous region with the fluid region. Then, the physical phenomena that take place in the adjacent regions are transferred from the interface, specifically the mass concentration tends to zero in both regions for $\alpha < 1$, due that the concentration in the interface also tends to zero (Figure **4**), before that in the adjacent regions. Now, for $\alpha = 1$, the interfacial concentration reaches a steady state with a value different to zero, causing a difference in concentration with respect to adjacent regions, which means that the mass transport is governed primarily by the diffusion process.

CONCLUSIONS

The numerical experiments show that the α parameter has an influence that facilitates the mass transfer of the porous region to the fluid region. In practice if we have a system with an α value close to one, is a challenge because it describes the limit case with a physical interpretation that both regions can be considered as identical, i.e., the resistance to mass transfer in the both regions are similar. The α parameter expresses the relation of the mass transfer resistances between the porous media and the homogeneous fluid, and in the present work is considered as a parameter which facilities the mass transfer of the porous region to the fluid region.

NOMENCLATURE

$a_{\sigma\beta}$	Interfacial area per unit volume of porous medium, m^2/m^3.
D_η	Mixture diffusion coefficient of the β-phase, m^2/s.
D_ω	Component in $y-$direction of the total dispersion tensor, m^2/s
C_η	Solute characteristic concentration in the external fluid, mol/m^3.
$\langle C_{A\beta} \rangle_\eta^\beta$	Intrinsic average concentration of species *A* in the β phase for the η homogeneous region, mol/m^3.
C_{Aj0}	Initial solute concentration in fluid, mol/m^3.

$\langle C_{A\beta}\rangle_{\omega}^{\beta}$ Intrinsic average concentration of species *A* in the $\beta-$ phase for the porous media ($\omega-$ region), mol/m^3.

L Thickness of the fluid channel, *m.*

K_{eq} Equilibrium coefficient for a linear adsorption (m).

Pe Peclet number, dimensionless.

U_j Solute dimensionless concentration in region $j(j=f,p)$.

$\langle v_{\beta x}\rangle_{\eta}^{\beta}$ Intrinsic average velocity of species *A* in the β-phase for the η, *m/s.*

Y Dimensionless position.

Subscrips

f,p, Fluid and Porous medium, respectively.

Greek Letters

δ Thickness of the homogeneous porous layer, *m*

$\varepsilon_{\beta\,\omega}$ Volume fraction of $\beta-$ phase for the $\omega-$ region

γ Dimensionless upscaled parameter

τ Dimensionless time.

REFERENCES

[1] Rangel-German ER, Kovscek AR. Water Infiltration in Fractured Systems: Experiments and Analytical Model. In: SPE Annual Technical Conference and Exhibition. New Orleans – Louisiana 2001. http://dx.doi.org/10.2118/71618-MS

[2] Kaya E, Zarrouk SJ, O'Sulliva. MJ. Reinjection in geothermal fields: A review of worldwide experience. Renewable and Sustainable Energy Reviews 2011; 15: 47-68. http://dx.doi.org/10.1016/j.rser.2010.07.032

[3] Beavers G, Joseph D. Boundary conditions at a naturally permeable Wall, J. Fluid Mech 1967; 30: 197- 207. http://dx.doi.org/10.1017/S0022112067001375

[4] Neale G, Nader W. Practical signicance of Brinkman´s extension of Darcy´s law: coupled parallel flows within a channel and a bounding porous medium. Can J Chem Energ 1974; 52: 475-478. http://dx.doi.org/10.1002/cjce.5450520407

[5] Vafai K, Tien CL. Boundary and inertia effects on flow and heat transfer in porous media. Int J Heat Mass Transfer 1981; 24: 195-203. http://dx.doi.org/10.1016/0017-9310(81)90027-2

[6] Haber S, Mauri R. Boundary conditions for Darcy´s flow through porous media. Int J Multiphase Flow 1983; 9: 561-574. http://dx.doi.org/10.1016/0301-9322(83)90018-6

[7] Poulikakos D, Kazmierczak M. Forced convection in a duct partially filled with a porous material. J Heat Transfer 1987; 109: 653-662. http://dx.doi.org/10.1115/1.3248138

[8] Prat M. On the boundary conditions at the macroscopic level. Transport in Porous Media 1989; 4: 259-280. http://dx.doi.org/10.1007/BF00138039

[9] Vafai K. Kim SJ. Fluid mechanics of the interface region between a porous medium and a fluid layer – an exact solution. Int J Heat Fluid Flow 1990; 11: 254-256. http://dx.doi.org/10.1016/0142-727X(90)90045-D

[10] Prat M. Some refinements concerning the boundary conditions at the macroscopic level. Transport in Porous Media 1992; 7: 147-161. http://dx.doi.org/10.1007/BF00647394

[11] Jang JY, Chen JL. Forced convection in a parallel plate channel partially filled with a high porosity medium. Int Commun Heat Mass Transfer 1992; 19: 263-273. http://dx.doi.org/10.1016/0735-1933(92)90037-I

[12] Sahraoui M, Kaviany M. Slip and no slip temperature boundary conditions at the interface of porous, plain media: Convection. Int Heat Mass Trans 1994; 37: 1029-1044. http://dx.doi.org/10.1016/0017-9310(94)90227-5

[13] Kuznetsov AV. Analytical investigation of the fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with a porous medium. Appl Sci Res 1996; 56: 53-67. http://dx.doi.org/10.1007/BF02282922

[14] Alazmi B, Vafai K. Analysis of fluid flow and heat transfer interfacial conditions between a porous medium and a fluid layer. Int J Heat Mass Trans 2001; 44: 1735-1749. http://dx.doi.org/10.1016/S0017-9310(00)00217-9

[15] Espinosa-Paredes G. Jump mass transfer for double emulsion systems. International Mathematical Forum 2 2007; (32): 1553-1570.

[16] Espinosa-Paredes G, Morales-Zárate E, Vázquez-Rodríguez A. Analytical Analysis for Mass Transfer in a Fractured Porous Medium. Petroleum Science and Technology 2013; 31: 2004-2012. http://dx.doi.org/10.1080/10916466.2011.557681

[17] Gwo JP, O'Brien R, Jardine PM. Mass transfer in structured porous media: Embedding mesoscale structure and microscale hydrodynamics in a two-region model. J Hydrologic 1998; 208: 204-222. http://dx.doi.org/10.1016/S0022-1694(98)00161-9

[18] Gibbs JW. The Collected Works of J. Willard Gibbs, vol. 1, Yale University Press, New Haven, Connecticut 1928.

[19] Slattery JC. Interfacial Transport Phenomena. Springer-Verlag, New-York 1990.

[20] Kaviany M. Principles of Heat Transfer in Porous Media, (Second Edition) Springer-Verlang, New York 1995. http://dx.doi.org/10.1007/978-1-4612-4254-3

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