Improved Determination of Apparent and Plastic Viscosity for Aqueous Solution of Drilling Fluid Additives

Dilip K. Rajak, Akhilendra K. Pathak and Chandan Guria*

Department of Petroleum Engineering, Indian School of Mines, Dhanbad 826 004, India

Abstract: Rheological behavior of drilling fluids is very complex. Exact determination of shear rates helps to predict apparent and plastic viscosity very accurately, which will help to monitor drilling operation efficiently. This paper deals with the improved estimation of shear rates of drilling fluids with varying rotor rotation using coaxial-cylinder Fann viscometer, which is based on generalized difference equation under purely steady, laminar and isothermal tangential fluid flow condition. Rotor rotating speeds and bob dial readings are the input variables for shear rate prediction. The proposed equation was used to calculate shear rates accurately (hence apparent, plastic viscosity and yield point) for several non-Newtonian fluids, mainly, aqueous suspension of bentonite, xanthan gum, poly anionic cellulose and carbomethoxy cellulose solution. Finally, the predicted consistency plots were compared with those which are obtained from the conventional method of estimating the rate of shear for drilling fluids.

Keywords: Shear rate, rheology, rotational viscometer, difference equation, plastic viscosity, apparent viscosity.

1. INTRODUCTION

Rheological behavior of drilling fluid additives (e.g., bentonite, xanthan gum, poly anionic cellulose and carbomethoxy cellulose and etc) is very complex due to non-Newtonian nature of the materials. The presence of fixed negative charges on bentonite (i.e., montmorillonite) surface yield a true colloidal suspension of aqueous bentonite solution. Similarly, water soluble polymeric drilling fluid additives behave as pseudo-plastic fluid behavior due to high degree of hydrophilic functional group substitution and degree of polymerization. The performance of oil well drilling was significantly affected by various rheological properties e.g., yield point, apparent viscosity, plastic viscosity and gel points [1-4]. Moreover, safety of the well was also restored while drilling by manipulating the exact values of these rheological properties. Fann viscometer is commonly used to measure the rheological properties of drilling fluid in oil fields as well as laboratories. It is a rotational coaxial-cylinder viscometer where drilling fluid is confined into the annular space between two cylinders (i.e., rotor: outer cylinder, and bob: inner cylinder), one of which is in motion (usually rotor) and other (usually bob) remains stationary after deflection while operation. Rotor rotations and bob deflections are the two measured readings which are directly used for rheological analysis of the test fluid. The torque exerted on the inner bob wall is measured directly from dial reading (i.e., bob deflection, θ) for given rotor rotation (i.e., rotor rpm), and it is converted easily into shear stress by neglecting end effect for known viscometer dimensions [3]. However, the major difficulty arises to predict the wall shear rate, which is mainly due to the non-uniform distribution of fluid flow in the concentric cylindrical annulus. Moreover, there is no exact method of calculating shear rate distribution using viscometer readings, unless the fluid model is assumed *a-priori*. Details of these techniques for shear rate estimation have been reviewed by [4-9]. To determine the rheological properties of drilling fluid, simple Newtonian approximation was assumed and the predicted shear rate is dependent on rotor rotation *only* [2-3]. This procedure is quite common and it is *mostly* practiced to determine the plastic viscosity and apparent viscosity of drilling fluids in oil fields and laboratories [1-3]. The major drawback of using this procedure is that the predicted shear rates are independent on the nature of the drilling fluids. In this regard, several studies have been made to predict the wall shear stress without assuming rheological model *a-priori* [9-10]. In this study, the work of Kumar and Guria [9] and Kumar *et al*. [10] was used to predict apparent viscosity and plastic viscosity of drilling fluid additives particularly aqueous suspension of bentonite, xanthan gum, poly anionic cellulose and carbomethoxy cellulose using Fann viscometer readings.

2. EXPERIMENTAL

2.1. Materials

In the present study, Fann 35 Viscometer (API RP 13B: Model 35) was used for rheological analysis of aqueous suspension of xanthan gum (XG), poly anionic cellulose (PAC), carboxy methyl cellulose (CMC) and

*Address correspondence to this author at the Department of Petroleum Engineering, Indian School of Mines, Dhanbad 826 004, India;
Tel: +91-326-2235411; Fax: +91-326-2296632;
E-mail: cguria.che@gmail.com

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bentonite separately at room temperature (i.e., 305K). The detail dimensions of above Fann 35 Viscometer were described by Bourgoyne *et al*., 1991. For consistent Fann 35 Viscometer readings, experiments were also repeated for several times with fresh suspensions. Aqueous suspension of *all* polymer and bentonite additives used for rheological analysis using Fann 35 viscometer was prepared as per API standards. Aqueous food grade XG solution with 0.28% and 0.85% concentrations were prepared using de-mineralized water by vigorous mixing and subsequent cooling. The concentration of 0.28%, 0.75% and 1.0% PAC solutions were prepared by adding PAC (specific gravity: 1.5) to the de-mineralized water with 4.0 % sodium chloride solution stirring. Similarly, 0.28% CMC solution was prepared by adding high viscosity grade CMC to the de-mineralized water with 4.0% sodium chloride solution under stirring at room temperature. Bentonite mud at high concentration exhibit Herschel-Bulkley fluid behavior with high yield stress. Bentonite powder was directly used for rheological analysis with following specifications: surface mean particle diameter-3.0 µm, loss on drying-3.0% and suspension pH-10.0 (suspension was prepared by dispersing 4.0 g of dried bentonite in 200 cm^3 de-mineralized water). Aqueous 6.0% bentonite suspension was prepared using standard Hamilton Beach commercial high-speed mixer (Model 550). It was mentioned that de-mineralized water with pH ~ 9.0 was used to prepare aqueous polymer solutions and pH of de-mineralized was adjusted by adding 2.0% aqueous sodium hydroxide solution.

3. WALL SHEAR RATE ESTIMATION

The generalized θ-component equation of motion for the coaxial rotational cylinder viscometer under purely steady, laminar and isothermal tangential fluid flow condition is given by the following differential equation [9-11].

$$\frac{1}{\mathrm{r}^2}\frac{d}{d\mathrm{r}}\left(\mathrm{r}^2\tau\right)=0 \tag{1}$$

Above equation is applicable to the fluids of any kinds with Newtonian as well as non-Newtonian behavior. Considering the slippage between two successive layers of the fluid in the annular space of the viscometer, the difference in shear rates at the bob and rotor wall is given by the following standard equation [5-7, 9,10, 12,13], i.e.,

$$\gamma_{\mathrm{r}1}-\gamma_{\mathrm{r}2}=2\tau_{\mathrm{r}1}\frac{\mathrm{d}\omega_2}{\mathrm{d}\tau_{\mathrm{r}1}} \tag{2}$$

where γ_{r1} = shear rate at the bob wall, γ_{r2} = shear rate at the rotor wall, τ_{r1} = shear stress at the bob wall and ω_2 = rotor rotation.

Above equation is quite general and can be applied to any non-Newtonian fluids (i.e., pseudoplastic and dilatants) and Herschel-Bulkley fluids (i.e., fluid with yield stress). The above equation (i.e., Eq. 2) can also be expressed in the form of difference equation and is written as:

$$2\tau\frac{\mathrm{d}\omega}{\mathrm{d}\tau}=\gamma(\tau)-\gamma\left(\mathrm{s}^2\tau\right) \tag{3}$$

where $\tau_{r1} = \tau$, τ_{r2} (Shear stress at the rotor wall) = $s^2\tau$ and s = ratio of bob to rotor radius (i.e., s = r_1/r_2 and 0 < s < 1.0).

Now expanding $\gamma\left(s^2\tau\right)$ in the form of Taylor series, one may obtain the following equation:

$$\begin{aligned}\gamma\left(s^2\tau\right)&=\gamma\left[\tau-\left(1-s^2\right)\tau\right]=\gamma(\tau)-\left(1-s^2\right)\tau\frac{d\gamma}{d\tau}\\&+\frac{\left(1-s^2\right)^2\tau^2}{2!}\frac{d^2\gamma}{d\tau^2}-\frac{\left(1-s^2\right)^3\tau^3}{3!}\frac{d^3\gamma}{d\tau^3}+\cdots\end{aligned} \tag{4}$$

Considering only the first two terms of the above series, the difference equation (i.e., Eq. 3) reduces to the following shear rate equation and is applicable to the Newtonian fluid *only* [3,9,10].

$$\gamma=\frac{2\omega}{\left(1-\mathrm{s}^2\right)} \tag{5}$$

Substituting the standard dimensions of Fann viscometer, one may obtain the following shear rate relation which is frequently used to calculate the rheological properties of drilling fluids for oil field applications[1-3] and is given by the following equation:

$$\gamma(\tau)=1.71N \tag{6}$$

where N = rotor rpm and ω = 2πN/60

It is noted that the above equation used frequently in the industry and drilling laboratories to predict apparent viscosity and plastic viscosity. Using equation of motion, continuity equation and Newtonian constitutive equation for standard rotational coaxial cylinder viscometer geometry at given rpm of rotor and corresponding dial reading of bob, above equation can also be derived separately for Newtonian fluids and the detail derivations are given by Bourgoyne *et al*. [3].

Neglecting the end effects in the rotational viscometer, wall shear stress for given dial reading was calculated according to Bourgoyne *et al*. [3], which is given by following equation i.e.,

$$\tau = \frac{k_1\theta}{2\pi r_1^2 h} \tag{7}$$

where k_1 = spring constant, θ = dial reading and h = height of bob.

Substituting the standard dimensions of Fann viscometer, one may obtained the following simplified equation for wall shear stress, i.e.,

$$\tau_w (Pa) = 0.51\theta \tag{8}$$

Using, Eqs. 6 and 8, one may obtained the following equations for plastic viscosity (PV), apparent viscosity (AV) and yield point (YP) i.e.,

$$\text{PV}(\text{cP}) = \theta_{600} - \theta_{300} \tag{9a}$$

$$\text{AV}(\text{cP}) = \frac{\theta_{600}}{2} \tag{9b}$$

and

$$\text{YP}\left(\frac{\text{lb}}{100\ \text{ft}^2}\right) = 2(\text{AV} - \text{PV}) \tag{9c}$$

Introducing first three terms of the Taylor series (i.e., Eq. 4) into the difference equation (i.e., Eq. 3), one will obtain the following shear rate equation:

$$\gamma(\tau) = -\beta^2\tau^{\beta+1}\int_0^\tau \omega(\text{x})\,\text{x}^{-(\beta+2)}\,\text{dx} \tag{10}$$

where $\beta = 2/\left(1-\text{s}^2\right)$ and x = shear stress (dummy variable).

Details of Eq. 10 are given in Appendix A. Substituting the power law type flow equation i.e., ω (x) = Kx^m and ω = 2πN/60 in Eq. 10, one may obtain the following rate of shear in terms of β, N and m.

$$\gamma(\tau) = \frac{\pi\beta^2}{30(\beta+1-\text{m})}\text{N} \tag{11}$$

Substituting the exact Fann Viscometer β value (i.e., β = 16.26), above equation reduces to

$$\gamma(\tau) = \frac{27.67}{17-\text{m}}\text{N} \tag{12}$$

In the above equation, 'm' is similar to the flow behavior index. It noted that Eq. 12 simplifies to Eq. 6 for m = 1 (i.e., Newtonian fluid). Using Eqs. 8 and 12, one may obtained the plastic viscosity in the following form i.e.,

$$\text{PV}(\text{Pas}) = \frac{0.51(\theta_{600} - \theta_{300})}{\gamma_{600} - \gamma_{300}} \tag{13}$$

On simplification, above equation reduces to

$$\text{PV}(\text{cP}) = \frac{(17-\text{m})}{16}(\theta_{600} - \theta_{300}) \tag{14}$$

For Newtonian fluids (i.e., m = 1), Eq. 14 reduces to Eq. 9a. The expression for apparent viscosity and yield point is also given by the following equations i.e.,

$$\text{AV}(\text{cP}) = \frac{(17-\text{m})}{32}\theta_{600} \tag{15}$$

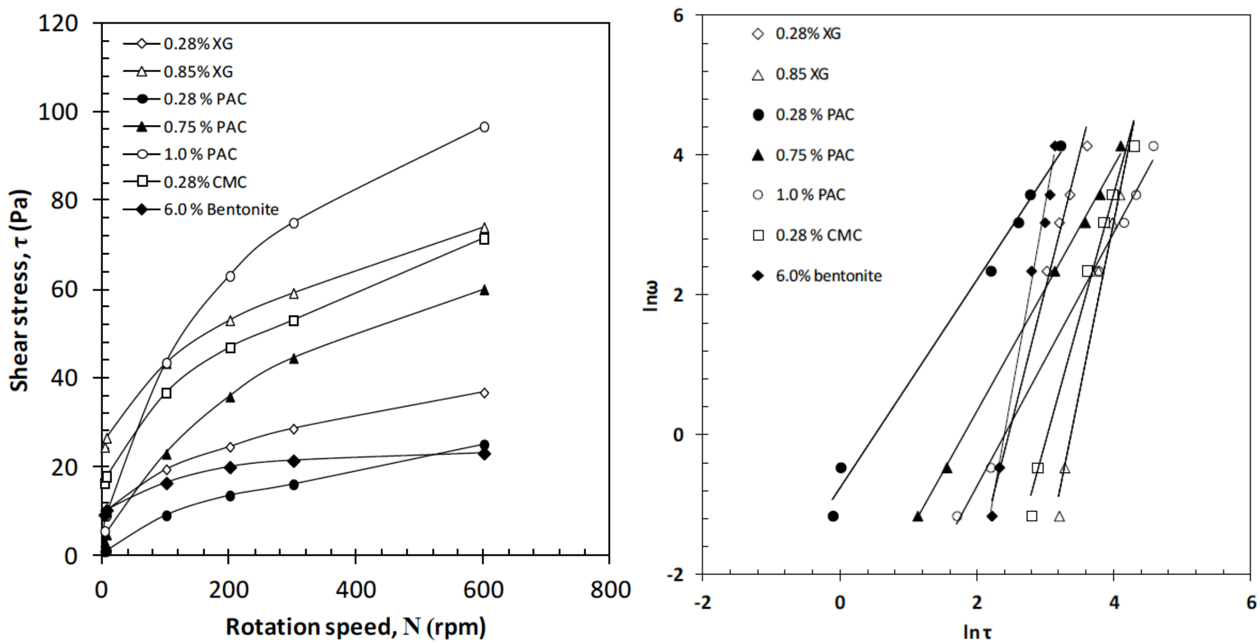
and

$$\text{YP}\left(\frac{\text{lb}}{100\ \text{ft}^2}\right) = \theta_{600} - 2(\theta_{600} - \theta_{300}) \tag{16}$$

Similarly, equation 15 also reduces to Eq. 9b after substituting m = 1 (i.e., Newtonian fluid approximation). It was observed that the Bingham yield point values were identical using Eq. 9c and 16. Therefore, Eqs. 14-16 constitute the proposed relations for calculating plastic viscosity, apparent viscosity and yield point using Fann viscometer readings.

4. RESULTS AND DISCUSSION

Fann viscometer readings (i.e., rotor rotations and bob deflections or dial readings) using aqueous polymer solutions of XG, PAC and CMC with varying composition and bentonite suspension were noted and details of wall shear stress *vs.* rotor rotation are shown in Figure **1a**. To evaluate 'm' and 'K' parameters of power-law type flow equation i.e., $\omega(\tau) = K\tau^m$, the best fit plots for lnω *vs*. lnτ were made for *all* the polymer and bentonite suspensions, and details are shown in Figure **1b**. Here, ω is calculated directly from rotor rotation (i.e., ω = 2πN/60) and τ is determined from Eq. 7 using bob dial readings. Details of the best fit equations with corresponding correlation coefficients (R^2), sum of the errors square (ΣQ^2) and the values of 'm' and 'K' for *all* experiments are listed in Table **1**. Now the values of 'm' for various polymer and bentonite suspension (Table **1**) were used to determine plastic viscosity, apparent viscosity and yield point



a b

Figure 1: a: Fann 35 Viscometer readings [τ_w (=0.51θ) vs. N] for aqueous xanthan gum, poly anionic cellulose, carboxy methyl cellulose solution and bentonite suspension.

b: Determination of 'K' and 'm' in $\omega(\tau) = K(\tau\text{-}\tau_0)^m$ for aqueous xanthan gum, poly anionic cellulose and carboxy methyl cellulose solution and bentonite suspension using Figure **1**.

Table 1: Evaluation of Parameter for Power Law Type $\omega(\tau) = K\tau^m$ Flow Equation

Description	lnω *vs.* lnτ	R^2	ΣQ^2	m	K, $rad.s^{-1}.Pa^{-n}$
0.28% XG	y = 3.8608x – 9.0350	0.9910	0.2059	3.86	0.0001
0.85 % XG	y = 4.8812x – 6.4890	0.9828	0.4098	4.88	6.90×10^{-8}
0.28% PAC	y = 1.4936x – 0.7729	0.9922	0.1865	1.49	0.4616
0.75% PAC	y = 1.7617x – 3.1843	0.9990	0.0240	1.76	0.0414
1.00% PAC	y = 1.8147x –0.3713	0.9960	0.1000	1.81	0.0126
0.28% CMC	y = 3.5674x –10.829	0.9851	0.3541	3.57	1.98×10^{-5}
6.0 % Bentonite	y = 5.5257x –13.318	0.9856	0.1381	5.53	1.64×10^{-6}
Average		0.9902	0.2026		

using Eqs. 14-16 and these results were compared with plastic viscosity, apparent viscosity and yield point obtained from Eq. 9. Details of the comparison of plastic viscosity, apparent viscosity and yield point are given in Table **2**. Though the Bingham yield point values for *all* test samples were identical using proposed and existing method, but plastic viscosity and apparent viscosity differ significantly by using proposed and existing method. The difference in plastic viscosity and apparent viscosity was mainly due to the deviation from ideal Newtonian behavior. Percentage error in viscosities were minimum when the values of 'm' is closer to 1.0 were as the deviation is maximum for the higher values of 'm'. It was found that the minimum deviation was observed for 0.28% PAC solution with 3.4 % whereas 6.0 % bentonite exhibit maximum deviation, which was equivalent to 28.0%.

CONCLUSIONS

A more accurate method of estimating plastic viscosity and apparent viscosity proposed for Non-Newtonian fluids using rotational narrow gap coaxial cylinder Fann Viscometer under purely steady, laminar

Table 2: Determination of Plastic Viscosity, Apparent Viscosity and Yield Point Using eq. 9 and Eqs. 14-16

Description	Eqs. 9a-9c			Eqs. 14-16		
	Plastic viscosity, cP	Apparent viscosity, cP	Yield point, lb/100 ft^2	Plastic viscosity, cP	Apparent viscosity, cP	Yield point, lb/100 ft^2
0.28% XG	15.7	35.3	39.2	12.9	28.9	39.2
0.85 % XG	23.6	70.6	94.0	17.9	53.5	94.0
0.28% PAC	15.7	23.6	15.7	15.2	22.8	15.7
0.75% PAC	31.3	58.8	55.0	29.8	56.0	55.0
1.00% PAC	41.1	94.1	106.0	39.2	89.3	106.0
0.28% CMC	35.3	69.6	68.6	29.6	58.4	68.6
6.0 % Bentonite	3.2	22.6	38.7	2.3	16.2	38.7

and isothermal tangential fluid flow condition. The proposed rheological analysis is quite general and can be applied to drilling fluids with non-Newtonian. Several drilling fluid additives (for example aqueous polymer solution of xanthan gum, poly anionic cellulose, carboxy methyl cellulose and bentonite suspension) are considered for rheological studies. The predicted plastic and apparent viscosities were compared with the results obtained from the formula given by Gatlin [1], Darley and Gray [2] and Bourgoyne *et al*. [3], and improved results were obtained using proposed equations.

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Appendix A: Details of shear rate estimation Eq. 10 [9,10]

The differential equation is obtained by introducing the first three terms of the Taylor series (i.e., Eq. 4) using Eq. 3 and is given by

$$2\tau\frac{d\omega}{d\tau}=(1-s^2)\tau\frac{df}{d\tau}-(1-s^2)^2\frac{\tau^2}{2}\frac{d^2f}{d\tau^2} \tag{A.1}$$

Substituting $\beta=\frac{2}{1-s^2}$ in above equation, we get the following simplified differential equation

$$\frac{d\omega}{d\tau}=\frac{1}{\beta}\frac{df}{d\tau}-\frac{\tau}{\beta^2}\frac{d^2f}{d\tau^2} \tag{A.2}$$

Again substituting $\frac{d\left(\tau\frac{df}{d\tau}\right)}{d\tau}=\frac{df}{d\tau}+\tau\frac{d^2f}{d\tau^2}$ in Eq. A.2 and integrating we get

$$\frac{df}{d\tau}-\left(\frac{\beta+1}{\tau}\right)f=-\beta^2\frac{\omega}{\tau} \tag{A.3}$$

Multiplying $e^{\int-\left(\frac{\beta+1}{\tau}\right)d\tau}$ on both sides of Eq. A.3 and subsequent integration leads to the following equation for shear rate

$$\gamma(\tau)=-\beta^2\tau^{\beta+1}\int_0^{\tau}\omega(x)x^{-(\beta+2)}dx \tag{10}$$

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