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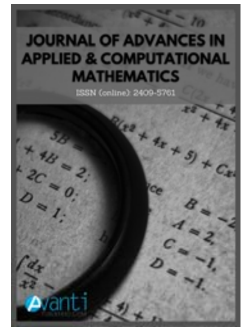

Soft Computing Under Uncertain Knowledge

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ABSTRACT

The early publications in the theory of fuzzy sets by Zadeh and Goguen show the intention to generalize the classical notion of a set and to accommodate the fuzziness that is contained in human language, namely human judgement, evaluation, and decisions. This article aims to show several approaches that allow effective treatment of uncertain, inaccurate, or unknown knowledge. On the one hand, a brief review of the theoretical background for these different paradigms is provided. On the other hand, the different extensions of soft sets are justified in the application to decision making. We pay special attention to applications in the medical sciences and provide a study case for biological signaling pathways.

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1. Introduction

Since its appearance in 1965, fuzzy set theory has evolved in various ways and applied in numerous disciplines. Some of the applications of the fuzzy set are in artificial intelligence, medicine, computer science, robotics, control engineering, decision theory, expert systems, logic, and pattern recognition. Related mathematical developments continue to improve to this day.

The use of imprecise or uncertain data is necessary for many real-life problems. The analysis of these problems requires the application of mathematical principles capable of capturing these characteristics. Traditional tools for modelling, reasoning, and computation are crisp, deterministic and precise. In this way, a statement in traditional logic can be true or false.

Fuzzy set theory was a paradigm shift in mathematics by allowing partial membership. Since Zadeh's seminal paper [105], the literature on fuzzy sets and their applications is extensive, including a number of successful generalisations. The purpose of fuzzy sets and related theories is to capture subjectivity, uncertainty, imprecision of valuations, etc. In this way, this theory efficiently handles practical cases in applied fields.

The theory of soft sets and their variations are one of the prominent extensions of fuzzy sets. The definition of soft sets is due to Molodtsov [68]. He also applies them in various areas and establishes the first main results. Molodtsov's idea is different from proposals of the same name of Pawlak [72] and Basu *et al.* [26]. However, these concepts share a common philosophy, where the concept of set differs from the classical concept of set theory.

The specification of parameters is not required in soft sets, as it supports all types of parameters. These parameters can be numbers, symbols, words, functions, etc. Thus, the definition of a soft set links the relevant attributes with the information of the universe elements [40].

Some relevant extensions are due to Maji *et al.* [63], Aktas and Çagman [9], Maji, Biswas, and Roy [60], who introduce fuzzy soft sets. Wang, Li, and Chen introduced hesitant fuzzy soft sets. N-soft sets were defined by Fatimah *et al.* [40]. Incomplete soft sets were initially developed thanks to Han *et al.* [47] and Zou and Xiao [111]. On the other hand, Jiang *et al.* [49] introduce an extended soft set theory on the basis of the description logics.

In addition to these soft set variants, we can also cite probabilistic soft sets [39, 110], soft binary relations [57], soft rough sets [44], fuzzy soft sets [1, 60], interval soft sets [108], soft fuzzy rough sets [106], intuitionistic fuzzy soft sets [61], interval-valued fuzzy soft sets [102], hesitant fuzzy soft sets [25], interval-valued intuitionistic hesitant fuzzy soft sets [75], interval-valued hesitant fuzzy soft sets [76], vague soft sets [95, 98], neutrosophic soft set [33], *ivnpiv*-neutrosophic soft sets [35], linguistic value soft sets, and so on. Ma *et al.* and Zhan and *et al.* wrote a survey of various hybrid models based on soft sets [59, 107]. The relationship between the theory of soft sets and hesitant fuzzy sets [21] and rough sets [14] is discussed in more detail below.

The organization of this paper is as follows. Section 2 motivates and provides the theoretical background about the extensions of soft sets. We then list the main properties and operations of soft sets, as well as other connected notions. Section 3 shows the application of extended soft set theory to decision-making in different areas. Section4 outlines a case study of soft sets for biological signaling pathways. Finally, Section 5 concludes our review.

2. Definitions Around Soft Sets

Preliminary notions of soft set theory are raised in this section. In each of the following subsections, the concepts associated with the different extensions of soft sets are introduced.

2.1. Complete Soft Sets and Incomplete Soft Sets

The description and terminology of soft sets are established with a universe of objects U and a universal set of parameters E.

Definition 1 (cf. Zadeh [105]) A fuzzy subset (FS) A of X is characterized by a function $\mu_A : X \to [0,1]$. For each $x \in X$, the number $\mu_A(x) \in [0,1]$ is called the degree of membership of x in the subset. It represents the degree of truth of the statement "x belongs to A".

FS(X) denotes the Zadeh's fuzzy subsets of X.

Definition 2 [cf. Molodtsov [68]] Let A be a subset of the set of parameters E. A pair (F, A) is called a soft set over U if F is a mapping $F : A \to 2^U$ from the set A to the set of all subsets of U.

Table 1: Tabular and Matrix Representation of a Soft Set (F, A)

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

	e_1	e_2	e_3	e_4
u_1	1	0	0	1
u_2	0	0	0	1
u_3	0	1	0	0
u_4	1	0	0	0
u_5	0	0	0	1

Therefore, a soft set can be understood as a parameterized family of subsets of U. For each of the elements e of the attribute set A, $F(e)$ is a subset of U, which is often called the set of approximate elements to e of the soft set (F, A). On the other hand, $F(e)$ can be thought of as a mapping $F(e) : U \to \{0,1\}$, so that $F(e)(u) = 1$ is equivalent to $u \in F(e)$.

This concept has been extensively researched and developed. Maji, Biswas, and Roy define concepts like soft equalities, intersections and unions of soft sets, and so on [63]. Feng and Li studied various types of soft subsets and their equality relations [42].

The universe of objects and the parameter sets are usually finite in practical applications. Naturally, soft sets can be represented either in matrix form or in tabular form [10, 103], where its cells values are either zeros or ones. Each row corresponds to an object in U and each column to a parameter in A. Example 1 shows a simple case of representation of a soft set.

Example 1 Let $U = \{u_1, \ldots, u_5\}$ and $A = E = \{e_1, e_2, e_3, e_4\}$. A soft set (F, A) over U can be described as $F(e_1) = \{u_1, u_4\}$, $F(e_2) = \{u_3\}$, $F(e_3) = \varnothing$, and $F(e_4) = \{u_1, u_2, u_5\}$; or also in tabular or matrix form (see Table **1**).

The definitions of restricted and extended intersections of soft sets are based on Ali *et al.* [23]—unions can be defined analogously. The following references provide the underlying interpretations of these definitions [22, 24, 89, 50].

Definition 3 [cf. Ali *et al.* [23]] Let (F, A) and (G, B) be two soft sets over the same universe U, such that $A \cap B \neq \varnothing$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_{\mathrm{R}} (G, B)$, and it is defined as $(F, A) \cap_{\mathrm{R}} (G, B) = (H, A \cap B)$, where $H(e) = F(e) \cap G(e)$ for all $e \in A \cap B$.

Definition 4 [cf. Ali *et al.* [23]] The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set $(H, C) = (F, A) \cap_{\mathrm{E}} (G, B)$, where $C = A \cup B$, and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & if\ e \in A \setminus B, \\ G(e), & if\ e \in B \setminus A, \\ F(e) \cap G(e), & if\ e \in A \cap B. \end{cases}$$

For better handling of uncertainty, Han *et al.* introduced incomplete soft sets. Soft sets are a particular case of incomplete soft sets.

Definition 5 [cf. Han *et al.* [47]] A pair (F, A) is an *incomplete soft set* over U when $A \subseteq E$ and $F : A \to \{0,1,*\}^U$, where $\{0,1,*\}^U$ is the set of all functions from U to $\{0,1,*\}$.

The $*$ symbol in this definition captures 'lack of information' [18]. That is, $F(e)(u) = *$ when it is not known whether u belongs to the subset of U approximated by e. The interpretation of $F(e)(u)$ for the other cases ($F(e)(u) = 1$ and $F(e)(u) = 0$) is identical to the soft sets.

Incomplete soft sets can be represented either by matrices or in tabular form in the same way as soft sets. In this case, the possible cell values are zeros, ones, and asterisks (for unknown values).

2.2. N-Soft Sets

N-soft sets are another generalization of the theory of soft sets. This new concept aims to achieve a parameterized description of the universe set that is neither binary nor continuous, as in the cases of soft sets and fuzzy soft sets, respectively. The foundation of N-soft sets lies in the finite-granular perception of attributes.

The description and terminology of N-soft sets are again established with a universe of objects U and a universal set of parameters E.

Definition 6 [cf. Fatimah *et al.* [40]] Let A be a subset of the set of parameters E. Let $R = \{0,1,\ldots,N-1\}$ be a set of ordered grades where $N \in \{2,3,\ldots\}$. A triple (F, A, N) is called a N-soft set on U if $F : A \to 2^{U \times R}$, with the property that for each $a \in A$ there exists a unique $(u, r_a) \in U \times R$ such that $(u, r_a) \in F(a)$, $u \in U$, and $r_a \in R$.

Starting from an attribute a, each object u receives exactly one evaluation of the space of evaluations R, namely the only r_a for which (u, r_a) belongs to $F(a)$. We can use a notation similar to soft sets, writing $F(a)(u) = r_a$ as a shortcut to $(u, r_a) \in F(a)$. We consider again that sets $U = u_i, i = 1,2,\ldots, p$ and $A = a_j, j = 1,2,\ldots,q$ are finite. Naturally, a N-soft set can also be presented in tabular form where r_{ij} means (u_i, r_{ij}) belongs to $F(a_j)$ or $F(a_j)(u_i)$ is equal to r_{ij}.

On the other hand, we can extend the previous definition to take into account incomplete information in the environment of N-soft sets and other related definitions [17, 18, 47, 77, 111]. These definitions also provide a complement to the incomplete fuzzy soft sets [58, 36].

Definition 7 [cf. Fatimah *et al.* [40]] Let $R = \{0,1,\ldots,N-1\}$ be a set of ordered grades where $N \in \{2,3,\ldots\}$. A triple (F, A, N) is called an incomplete N-soft set on U if $F : A \to 2^{U \times R}$, with the property that for each $a \in A$ and there exists at most one $(u, r_a) \in U \times R$ such that $(u, r_a) \in F(a)$, $u \in U$, and $r_a \in R$.

Definition 8 [cf. Fatimah *et al.* [40]] Let (F,A,N) be an N-soft set. The normalized N-soft set from (F,A,N) is an N-soft set (F^0,Q,N) such that for all $a_i \in A, u_i \in U$, $F^0(a_j)(u_i) = F(a_j)(u_i) - m$, where $m = min\, r_{ij}$ is in the tabular representation of the original (F,A,N) and $Q = \{1,2,\ldots,q\}$.

Definition 9 [cf. Fatimah *et al.* [40]] Two N-soft sets (F,A,N) and (F',A',N') are equal over the same universal U if and only if $F = F', A = A'$, and $N = N'$. This is denoted by $(F,A,N) = (F',A',N')$.

Definition 10 [cf. Fatimah *et al.* [40]] Two N-soft sets (F,A,N) and (G,A',N) on U are equivalent under normalization if $(F^0,Q,N) = (G^0,Q',N)$.

The union and intersection of sets are always concepts to be developed and studied. Based on the ideas of Ali *et al.* [23], Fatimah *et al.* defined the restricted and extended intersection and union of N-soft sets. In both cases, the finiteness assumption is not imposed.

Definition 11 [cf. Fatimah *et al.* [40]] Let U be a fixed universe of objects. The restricted intersection of (F,A,N_1) and (G,B,N_2) is denoted by $(F,A,N_1) \cap_{\mathsf{R}} (G,B,N_2)$, and is defined as $(H, A\cap B, min(N_1,N_2))$, where for all $a \in A \cap B$ and $u \in U$,

$$(u,r_a) \in H(a) \Leftrightarrow r_a = min(r_a^1, r_a^2), if\, (u,r_a^1) \in F(a)$$

$$and (u,r_a^2) \in G(a)$$

Definition 12 [cf. Fatimah *et al.* [40]] Let U be a fixed universe of objects. The extended intersection of (F,A,N_1) and (G,B,N_2) is denoted by $(F,A,N_1) \cap_{\mathsf{E}} (G,B,N_2)$, and is defined as $(J, A\cup B, max(N_1,N_2))$, where:

$$J(a) = \begin{cases} F(a), if\ \ a \in A \setminus B, \\ G(a), if\ \ a \in B \setminus A, \\ (u,r_a)\, suchthat r_a = min(r_a^1, r_a^2), where \\ \qquad (u,r_a^1) \in F(a) and (u,r_a^2) \in G(a). \\ \end{cases}$$

2.3. Hesitant Fuzzy Sets

The fuzzy hesitant set allows handling situations where there are some difficulties in determining whether an element belongs to a set due to hesitancy between different values [97]. Initially, we introduce the definitions of hesitant fuzzy sets and typical hesitant fuzzy sets.

To deal with such sets, we consider a fixed, non-empty set of alternatives X. Additionally, we rescue the notation used by Alcantud and Torra [28]: the set of all subsets of X is denoted as $P(X)$, the set of non-empty subsets of X is denoted as $P^*(X)$, the set of non-empty finite subsets of X is denoted as $F^*(X)$, and for each $N \in \mathbb{N}$, the set of non-empty subsets of X with N or fewer elements is denoted as $F_N^*(X)$.

Definition 13 [cf. Xia and Xu [97]] A hesitant fuzzy element (HFE) is a non-empty, finite subset of $[0,1]$. The set of HFEs is denoted by $F^*([0,1])$.

Hesitant fuzzy elements are usually written as $h = \{h^1,\ldots,h^{l_h}\}$ where $h^1 < \ldots < h^{l_h}$ and the cardinality of an HFE h is written as l_h. The complete and empty HFEs are $h = \{1\}$ and $h = \{0\}$, respectively.

Definition 14 [cf. Torra [92]] A *hesitant fuzzy set* (HFS) on X is a function $h_M : X \to P([0,1])$. The set of HFSs on X is denoted as **HFS**(X).

Definition 15 [cf. Bedregal *et al.* [28]]A *typical hesitant fuzzy set* (THFS) on X is $h_M : X \to F^*([0,1])$. The set of THFSs on X is denoted as HFS^t(X).

A set of membership values for each element of X is defined in each hesitant fuzzy set over X, and that set of values will be finite and non-empty in the case of a typical hesitant fuzzy set. Therefore, a hesitant fuzzy set corresponds to the set of possible membership values of a typical hesitant fuzzy set in an alternative. Thus, in each alternative, we must make at least one assessment since the corresponding codomains in these definitions are $P^*([0,1])$ and $F^*([0,1])$.

Any fuzzy subset on X with membership function $\mu_M : X \to [0,1]$ such that $\mu_M(x) = M_x$ can be identified with the typical hesitant fuzzy set h_M described as $M = \{(x, h_M(x)) \mid x \in X, h_M(x) = \{M_x\}\}$ [28]. Stated another way, FS is THFS with the association defined above.

For a typical fuzzy set h_M in X, we can write $h_M(x) = \{h_M^1(x), \ldots, h_M^{l_M(x)}(x)\}$, where $h_M^1(x) < \mathrm{K} < h_M^{l_M(x)}(x)$ and $l_M(x) = | h_M(x) |$ is the cardinality of the HFE $h_M(x)$.

The union of two HFSs h_1 and h_2 on X, defined by Torra and Narukawa [92, 94] and denoted as $h_1 \cup h_2$, for $x \in X$ is:

$$(h_1 \cup h_2)(x) = \{h \in h_1(x) \cup h_2(x) : h \max\{\inf h_1(x), \inf h_2(x)\}\}.$$

In the case of typical hesitant fuzzy sets,

$$(h_1 \cup h_2)(x) = \{h \in h_1(x) \cup h_2(x) : h \max\{h_1^1(x), h_2^1(x)\}\}.$$

This definition corresponds to the classical fuzzy set theory definition of union when h_1 and h_2 are FSs.

2.4. Rough Sets

The rough set theory deals with vagueness from a different perspective [14]. Initially, some notation and definitions are introduced.

The universe of discourse U is the set of options, which must be non-empty and finite. Given a set U, its power set is denoted as $\mathrm{P}(U)$ and its cardinality is denoted as $|U|$. The parameter space E_U or E is the set of all parameters—associated with options in U. A soft universe is a pair (U, E).

Given an equivalence relation ρ on the set U, $[u]_\rho$ represents the equivalence class of an element u of U under the equivalence relation ρ and U/ρ is the quotient set of the set U by the equivalence relation ρ.

On the other hand, $\mathrm{S}^E(U)$ is the collection of all soft sets over U with parameter sets of E. $\mathrm{S}_A(U)$ is the collection of all soft sets over U with fixed parameter set A, which is a subset of E.

Definition 16 [cf. Feng *et al.* [44]] Let $S = (F, A)$ be a soft set over U. If $\Upsilon_{a \in A} F(a) = U$, then S is said to be a full soft set.

Definition 17 [cf. Feng *et al.* [44]] A soft set $S = (F, A)$ over U is called a partition soft set if $\{F(a) : a \in A\}$ forms a partition of U.

The theory of rough sets deals with vagueness from a different point of view. Rough sets focus on the granulation of the universe, which is usually produced by indiscernibility. The granularity of rough sets facilitates flexible and feasible computing approaches.

For rough sets, in Pawlak's classical models, the approximation space is formed by the universe of discourse and an equivalence relation on this set. The approximation space produces a granulation structure in which any subset X of the universe of discourse can be written as a function of subsets called lower and upper approximations of X.

Definition 18 [cf. Pawlak [74]] Let ρ be an equivalence relation on the universe U. Let X be a subset of U. The ordered pair (U, ρ) defines a Pawlak approximation space. The lower approximation of X is denoted as

$$\underline{\rho}(X) = \{u \in U \mid [u]_\rho \subseteq X\},$$

and the upper approximation of X is denoted as

$$\overline{\rho}(X) = \{u \in U \mid [u]_\rho \cap X \neq \emptyset\}.$$

A set X is definable if the lower and upper approximations are equal, i.e., $\underline{\rho}(X) = \overline{\rho}(X)$; otherwise, the set X is a rough set.

The algebraic structure of the universe of discourse allows recourse to reinforced forms of equivalence relations. An advantage of rough sets is that they can handle inconsistent sets of examples; that is, objects that are not discernible through condition attributes but discernible through decision attributes [73].

3. Decision Making Under Uncertainty

The use of soft sets in real-life problems is abundant and some references will be included for the different soft set extensions.

3.0.1. Decision Making with Pure and Incomplete Soft Sets

The basic reference for decision making in soft sets is due to Maji *et al.* [62]. They establish a choice criterion in decision making which is to maximize the choice value. From the matrix form of the soft set, the application of this criterion is straightforward. Let $T = (t_{ij})_{k \times l}$ be the soft set matrix, where k and l are the cardinals of the sets U and A, respectively. Then we define the *choice value* of each object h_i in U as $c_i = \sum_j t_{ij}$. Finally, the chosen choice will be the object h_k which verifies $c_k = \max_i c_i$.

In the case of soft sets and fuzzy soft sets, the analysis under incomplete information was initiated by Zou and Xiao [111]. In their proposal, all possible choice values in each object are calculated, and the *decision values* d_i are calculated by the weighted-average method. This average weight is computed from the complete information available. Some examples of indicators that can be used to prioritize alternatives are: (1) take the choice value $c_{i(0)}$ with null assignment for all unknown values, (2) take the choice value $c_{i(1)}$ with the unit assignment for all unknown values, and (3) take the mean of the previous choice values (d_{i-p}).

Qin *et al.* [77] consider a different way to fill in the unknown data in an incomplete soft set. In their approach, they complete the soft set based on the association between different parameters. This method is appropriate on the assumption that there is an association or correlation between the parameters.

Finally, we highlight the algorithms proposed by Alcantud and Santos-García [17, 18], which do not start from information on the relationship between objects and parameters. Their method consists of completing an incomplete soft set in all possible ways and studying the results in each of these complete soft sets.

3.0.2. Decision Making with N-Soft Sets

In this subsection, some decision-making procedures for N-soft sets are presented. Based on the relevance of grades in real-world situations, N-soft sets can include parameterized descriptions of the universe of alternatives that depend on a certain (finite) number of grades [40]. A matrix of grades can be defined simply and naturally for an N-soft set (F, A, N):

$$(r_{ij})_{p\times q} = \begin{pmatrix} r_{11} & r_{12} & \ldots & r_{1q} \\ \ldots & \ldots & \ldots & \ldots \\ r_{p1} & r_{p2} & \ldots & r_{pq} \end{pmatrix}$$

This decision-making procedure proposed by Fatimah *et al.* [40] is a generalization of the procedure for soft sets without parameter reduction proposed by Maji *et al.* [64]. The different options are ranked according to their extended choice values or their extended weight choice values. The first of the algorithms is shown below:

Algorithm 1 The algorithm of extended choice values.

1. Input objects and attributes: $U = \{u_1, \ldots, u_p\}$ and $A = \{a_1, \ldots, a_q\}$.

2. Input the N-soft set (F, A, N), where $R = \{0, 1, \ldots, N-1\}$, and N belongs to $\{2, 3, \ldots\}$, so that for all u_i in U, a_j in A, and there exists a unique r_{ij} in R.

3. For each u_i, calculate its extended choice value $\sigma_i = \sum_{j=1}^{q} r_{ij}$.

4. Find all indices k for which $\sigma_k = max_{i=1,\ldots,p}\ \sigma_i$.

5. The solution is any u_k from the previous step.

3.0.3. Decision Making with Rough and Hesitant Fuzzy Sets

Regarding rough sets, Feng [44] proposed a soft rough set-based scheme for supporting multicriteria group decision-making.

In fuzzy set theory, the concept of the extension principle that Zadeh [105] first introduced is important [29, 71, 101]. The extension principles allow us to calculate an (approximate) functional dependence between variables even in cases where you have approximate knowledge of the argument of a given precise mapping. In this way, arithmetic operations for fuzzy numbers can be defined by applying the extension principle to the standard operations for real numbers.

Extension principles are therefore directly applicable in decision making since they enable the researcher to implement decision-making mechanisms designed for hesitant fuzzy sets when the input data are given in other formats.

Feng *et al.* [43] investigate decision-making problems by using fuzzy soft sets [96]. Thanks to their results, among others [94, 100], hesitant fuzzy soft set-based decision-making problems can be successfully addressed.

3.1. Decision Making in Medical Sciences

In this subsection, we delve into the problem of applying soft sets and other extensions in the practice of decision-making in medicine.

With regard to diagnosis, Sanchez [82] is one of the pioneers in the use of fuzzy logic in medical diagnostics. Since then, many authors have worked on decision-making applied to medical sciences. Pawlak *et al.* [74] addressed the classification of patients after vagotomy for duodenal ulcers by using rough sets. A prediction system according to which the risk of Glaucoma can be assessed was developed by Alcantud *et al.* [19]. Çelik and Yamak [30] proposed fuzzy soft sets in medical diagnosis, Stefanowski and Slowinski [91] show applications of rough sets to identify the causal relevancy of particular pre-therapy attributes. Alcantud *et al.* [19] carried out an analysis of survival for lung cancer resections cases with fuzzy and soft set theory in decision making. Yuksel *et al.* [104] used soft sets to establish prostate cancer risk,

Slowinski [90] applies rough set theory for the analysis of duodenal ulcers. For new patients with duodenal ulcers due to herpes simplex virus, he achieves an assessment in treatment.

Santos-Buitrago *et al.* [86] review artificial intelligence methods for melanoma modelling. Santos-Buitrago *et al.* [84] study the molecular signalling pathways with incomplete soft sets. Fu *et al.* [46] use hesitant fuzzy sets for an initial evaluation method of prostatic hyperplasia symptoms. Lashari and Ibrahim [55] used soft sets for medical image classification.

Chetia and Das [31] use the interval-valued fuzzy soft set environment for medical diagnosis. Das and Kar [32] apply soft computing to viral fever-related diagnosis. Kirisci [53] proposes a soft set and medical decision-making application. Some problems related to the diagnosis of disease are solved by other authors [27, 65].

4. Study Case: Soft Sets for Biological Signaling Pathways

The study of biological systems has great importance [37]. Given the size and complexity of the cellular signaling networks, it has become necessary to develop predictive mathematical models to understand the system behavior of these networks and to predict higher-order functions that can be validated by experiments [85]. Figure **1** illustrates the main signaling pathways that influence uveal melanoma [83].

There exist numerous approaches to signal transduction processes, including symbolic modeling of cellular adaptation [37]. The use of formal methods for computational systems biology eases the analysis of cellular models and the establishment of the causes and consequences of certain cellular situations associated with diseases.

Pathway Logic is a symbolic systems biology approach for modeling and analyzing biological processes, such as signal transduction. Pathway Logic models are represented and analyzed using Maude, a formal system based on rewriting logic [54]. Pathway Logic allows researchers to develop abstract qualitative models, even quantitative and probabilistic models, of signaling processes that can be used as the basis for analysis by powerful tools to study a wide range of questions [87].

Figure **2** shows a general view of Pathway Logic Assistant, which is a Java software that implements the Pathway Logic vision. It shows the Petri net representation of a signaling pathway. Rectangles are transitions (biochemical reactions), and ovals are occurrences (biological entities) in which the initial occurrences are dark. The reactants of a rule are the occurrences connected to the rule by arrows from the occurrence to the rule. The products of a rule are the occurrences connected to the rule by arrows from the rule to the occurrence. Dashed arrows indicate an occurrence that is both input and output. For example, the reaction/rule 1229c appears in Figure 2. In this rule, Jak1 protein (in the cytoplasm) and Gp130 transmembrane protein (at GP130C location) intervene as reactants. The result of this reaction is that Gp130 protein is unchanged, and Jak1 moves from cytoplasm to GP130C location [88].

Pathway Logic models are structured in four layers: sorts and operations, components, rules, and queries. The *sorts* and *operations* layer declares the main sorts and sub sort relations, the logical analogue to ontology. The sorts of entities include Chemical, Protein, Complex, Location (cellular compartments), and Cell. These are all sub sorts of the Soup sort that represents unordered multisets of entities.

Rewrite rules detail the behavior of cell components depending on biological contexts and modification states. Each rule represents an action in a biological process, such as intra/intercellular signaling reactions or metabolic reactions. For example, we can say that if in the location CLc (that corresponds to the cytoplasm), a protein Ikke is found in an activated form and a protein of the family Akts is found, then the protein Akts will be phosphorylated on S473 and T308:

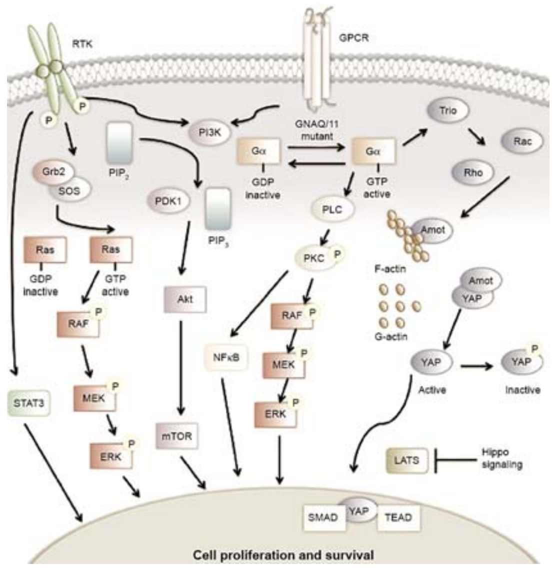

Figure 1: Signaling pathways in uveal melanoma. Source: Santos-Buitrago *et al.* [86].

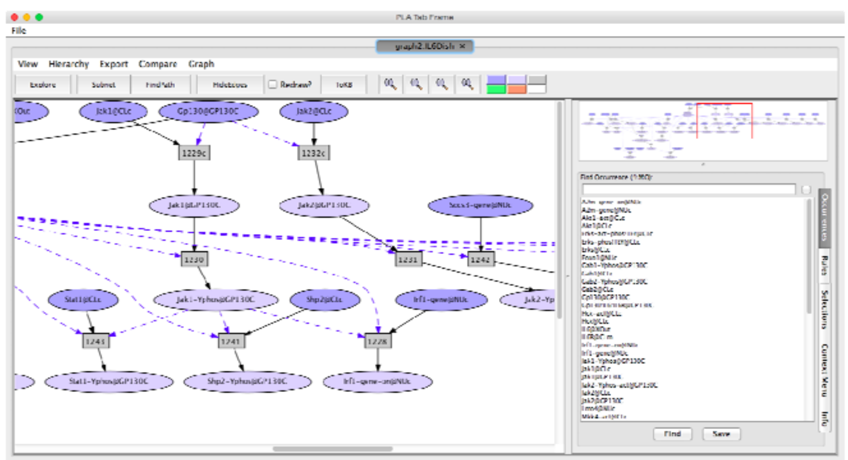

Figure 2: A general view of a signaling pathway using Pathway Logic Assistant (cf. Santos-García *et al.* [88]).

rl[1598c.Akts.by.Ikke]: {CLc | clc [Ikke - act] Akts} => {CLc | clc [Ikke - act] [Akts - phos(S 473) phos(T 308)]}

where the variable clc stands for any other element that might appear in the corresponding location (see Figure **3** for a schematic representation of this rule).

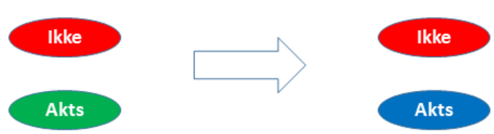

Figure 3: Schematic representation of rule 1598c.Akts.by. Ikke. The blue color of the Akts protein indicates that it is phosphorylated at the next state.

In a classical Pathway Logic system, all possible rewriting rules of the signaling path are considered [80]. Now we apply the decision-making procedure under incomplete information according to Zou and Xiao [111]. In order to choose the most appropriate rule to be executed, we need to extract the incomplete soft set information and manipulate it. This strategy computes all the reachable terms from the current term, takes the information of the soft set attributes, and places it into a matricial representation of the incomplete soft set SoftSet, which is defined as a list of lists of values [84].

In order to apply our strategy, we require terms and rules to be extended with an incomplete soft set. In Maude, the SOFTSET module defines attributes of incomplete soft set objects as a set of pairs of the form [Att = V], where Att is the attribute name, and V is its value, which can take the values 0, 1, and *. In this way, different attributes, when put together and separated by commas, produce a term of sort AttSet of the form [Att1 = V1], ..., [Attn = Vn].

Once we have defined attributes of incomplete soft sets, we can extend terms and rules to deal with them. We transform each rewrite rule rl T => T' . (respectively, conditional rules crl T => T' if COND ., for COND a condition), for T and T' terms of the given system, as rl T Atts => T' Atts' . (respectively crl T Atts => T' Atts' if COND .), with Atts the attributes of the incomplete soft set before applying the rule and Atts' the transformed attributes of the incomplete soft set. Note how the values of the attributes in the incomplete soft set can be modified in each rewrite rule.

In this way, an application of logic modeling with rewriting logic and soft set theory is defined. This approach to decision-making with soft sets offers a strategy that complements standard strategies. A metalevel strategy is implemented to control and guide the rewriting process of the Maude rewriting engine. With this case study, logical-mathematical methods capture imprecision, vagueness, and uncertainty in the available data.

5. Conclusions

The early publications in the theory of fuzzy sets by Zadeh and Goguen show the intention to generalize the classical notion of a set and to accommodate the fuzziness that is contained in human language, namely human judgement, evaluation, and decisions.

This paper aims to show several approaches that allow for effective treatment of uncertain, inaccurate, or unknown knowledge. There are many perspectives on the problem of how to understand and manipulate imperfect knowledge. Different extensions of soft sets are justified in the application to decision making, paying special attention to applications in the medical sciences.

The main drawback in the use of these soft computing techniques is the difficulty in determining the type of sets to use according to the type of inaccuracy and characteristics of the data available for each problem.

Standard soft set theory is based on a binary description of the universe of objects [64], which many researchers consider to be too simple in many real-life decision-making situations. For this reason, numerous expanded notions and approaches have been incorporated into soft computing [8, 10-12, 34, 41, 48, 52, 69, 109].

Along with the studies by Rodríguez *et al.* [81] and Xu [99], there are many other recent papers on the theoretical approaches on hesitant fuzzy sets and their practical applications, which show the importance of hesitant fuzzy elements and sets. Alcantud introduced an approach to analyze projects characterized by hesitant fuzzy sets by relating hesitant fuzzy sets to other soft computing models [13]. Other papers on hesitant sets are [2, 15, 16, 20, 66, 67, 70]

The theory of rough sets deals with vagueness from a different point of view [14]. Rough sets do not aim to obtain approximate descriptions based on the parameterization of the universe of discourse as in the case of soft sets and their generalizations. Rough sets focus on the granulation of the universe, which is usually produced by indiscernibility.

Nowadays, numerous new extensions in soft computing and applications in decision making are still being developed [3-7, 38, 45, 51, 56, 78, 79, 93].

A case study on biological pathways has been summarized. Our ultimate aim is to automatically determine the rules that are most appropriate and adjusted to reality in dynamic systems using decision-making with incomplete soft sets.

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