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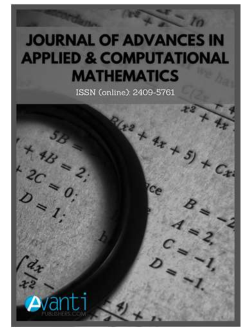

Brake Power Required to Avoid Vehicular Obstruction with a Steadfast Obstacle: A Probabilistic Approach

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ABSTRACT

A probabilistic approach is applied to assess the level of the available brake power needed to avoid obstruction with a steadfast obstacle suddenly detected in front of a moving vehicle. The obtained results can be used to establish, on the design stage, the level of this power for an extraordinary situation of the type in question and for an acceptable (in effect, never-zero) probability of an accident.

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1. Introduction

Whether electromagnetic, frictional or hydraulic, the automotive brake system is supposed to absorb the energy from a moving vehicle and is, therefore, a must for all types of vehicles [1, 2]. The characteristics of a brake system include, as is known, many characteristics, such as peak force, continuous power dissipation, fade, smoothness, power, pedal feel, drag, durability, weight, and even the level of noise. However, this analysis is limited to the role and level of the required brake system power.

As is known, two major phases of the stopping process are being distinguished [1-4]: the first one, the pre-braking phase, includes the driver's perception (decision making) time and the reaction time, and is characterized by the more or less constant speed of the vehicle; the second one, when actual braking takes place, is characterized by a constant deceleration, and it is at this phase when an adequate brake power is important. In the analysis that follows, a probabilistic approach is applied to assess the level of this power needed to avoid obstruction with a suddenly detected steadfast obstacle in front of the moving vehicle. A situation when the only way to avoid an accident is by using brakes (see, e.g., Fig. **1**) is addressed. The objective of the analysis is to establish the probability of the possible obstruction. The obtained results can be used to determine this probability when selecting the power of the brake system. The analysis is an extension and a modification of the approach considered earlier for a spacecraft landing on a planet [5].

Figure 1: Steadfast obstacle on the road suddenly detected in front of the moving vehicle.

2. Analysis

At the beginning of the braking time of the duration t, the vehicle's kinetic energy $K = \frac{MV_0^2}{2}$ is expected to assure the safe outcome of such an extraordinary situation and be totally absorbed during this time. This means that the system should possess the power $P = \frac{MV_0^2}{2t}$ to do its job. Here M is the vehicle's mass and V_0 is its speed at the beginning of the braking time. This time can be found for the given braking distance S and the initial speed V_0 as $t = \frac{2S}{V_0}$, and therefore

$$P = \frac{MV_0^2}{2t} = \frac{MV_0^3}{4S}. \tag{1}$$

No wonder the existing safety regulations require that in hazardous situations of the type in question, the initial driving speed V_0 should be kept as low as possible.

The mass of the car and its initial velocity are usually known reasonably well compared to the available braking distance S even if radars ("radio detection and ranging") and/or lidars (optical radars: "laser imaging, detection, and ranging") are employed. In our analysis, this distance is treated as a random variable. We assume that it is distributed in accordance with the Rayleigh law (Fig. **2**):

$$f_s(s) = \frac{s}{D}\exp\left(-\frac{s^2}{2D}\right), \tag{2}$$

where the notation

$$D = \sigma^2 = \frac{2}{\pi}\bar{s}^2 \tag{3}$$

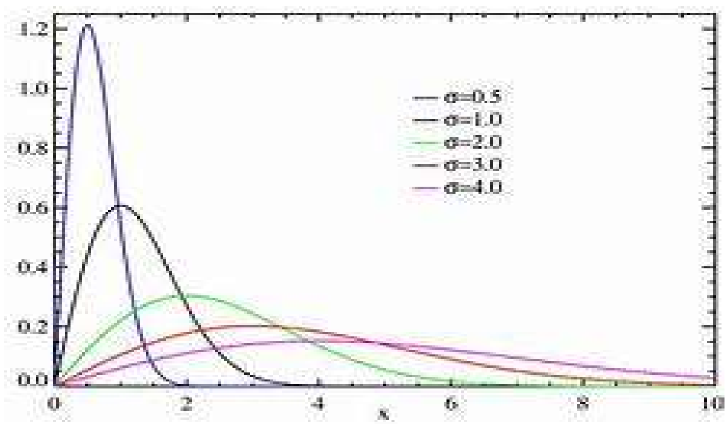

Figure 2: Rayleigh distribution (here σ is the mode, i.e., the most likely value of the random variable S)

is used. Here σ is the mode (the most likely value) of the distance S and $\bar{s}$ is its mean value. The variance D_* of the distribution (2) is related to the D value as $D_* = \frac{4-\pi}{2}D.$ The rationale behind such an assumption is that the sought probabilities should be zero for both zero and very large braking distances S, that low distances S are much more likely, and, as far as this analysis is concerned, are of greater interest than large distances. Because of that, the physically meaningful probability density distribution function $f_s(s)$ for the distance S should be heavily skewed towards low S values. Rayleigh distribution is the simplest one that possesses these physically meaningful properties.

Treating the random braking power P as a non-random function of the random variable S (see, e.g., [5]), we obtain the following expression for the probability density function of this power:

$$f_P(p) = f_s(s)\frac{ds}{dp} = \frac{M^2V_0^6}{4Dp^3}\exp\left(-\frac{M^2V_0^6}{8Dp^2}\right). \tag{4}$$

The braking power should be above a certain non-random level p_* to avoid obstruction. Therefore, the probability

$$Q = Q(P \succ p_*) = 1 - \int_0^{p_*} f_p(p)dp = 1 - \int_0^{p_*} \frac{M^2 V_0^6}{4Dp^3} \exp\left(-\frac{M^2 V_0^6}{8Dp^2}\right) dp = 1 - \exp\left(-\frac{M^2 V_0^6}{8Dp_*^2}\right) = 1 - \exp\left(-\frac{M^2 V_0^6}{8\sigma^2 p_*^2}\right) \quad (5)$$

could be viewed as the probability of the inability of the braking system to do its job, i.e., as the probability of its functional failure. As evident from (5), the probability of hitting the obstacle is lower for a lower mass M of the vehicle, for a lower initial speed V_0, for a higher most likely available distance σ, and a higher available power of the braking system. The formula (5) also indicates that the initial speed V_0 is particularly critical.

We want to emphasize that it is a simplified treatment of the problem and aims to demonstrate the usefulness of the probabilistic approach. Many important factors are not taken into account here: the road and tire conditions, the role of the human factor, the friction between the tires and the road (as is known, for rubber tires, the coefficient of friction decreases as the mass of the car increases), rubber temperature, weather conditions, air drag, etc.

3. Numerical Example

Let, e.g., the speed at the beginning of the braking distance for a heavy truck that weighs 30,000 lb =13,608kg (its mass is $M = 1387.2 kgxs^2 xm^{-1}$) and whose braking system's horse-power is $p_* = 1300hp = 97500kgm/s,$ is $V_0 = 50mph = 22.351m/s$. Then the formula (5) yields:

$$Q_1 = 1 - \exp\left(-\frac{1387.2^2 x 22.351^6}{8x97500^2 \sigma^2}\right) = 1 - \exp\left[-\left(\frac{56.1669}{\sigma}\right)^2\right]$$

If the initial velocity is only $V_0 = 40mph = 17.880m/s,$ then

$$Q_2 = 1 - \exp\left(-\frac{1387.2^2 x 17.880^6}{8x97500^2 \sigma^2}\right) = 1 - \exp\left[-\left(\frac{28.7536}{\sigma}\right)^2\right]$$

The calculated data for different most likely braking distances are as follows:

σ, m	50	75	100	150	200	250	300	350
Q_1	0.7169	0.4293	0.2706	0.1308	0.0758	0.0492	0.0344	0.0254
Q_2	0.2816	0.1367	0.0794	0.0361	0.0205	0.0131	0.00914	0.00673

As evident from these data, by slowing down the car in anticipation of a hazardous situation, one could dramatically reduce the accident's likelihood.

4. Conclusion

A probabilistic approach is applied to assess the required available brake power to avoid an automotive vehicle's obstruction with a suddenly detected steadfast obstacle. The obtained results can be used to establish this power to possibly avoid obstruction in off-normal situations of the type in question.

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