Identities with Generalized Derivations and Automorphisms on Semiprime Rings

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Abstract: In this paper we prove some results which extend Theorem 4, Theorem 10 and Theorem 11 of Vukman [13] and proposition 2.3 of Thaheem and Samman [10].

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1. INTRODUCTION

Throughout the paper R will denote an associative ring with centre $Z(R)$. Recall that R is prime if for any $a,b \in R$, $aRb = \{0\}$ implies that either $a = 0$ or $b = 0$ and is semiprime if for any $a \in R$, $aRa = \{0\}$ implies that $a = 0$. A ring R is said to be a 2 -torsion free if $2x = 0$ for $x \in R$ implies that $x = 0$. We shall write for any pair of elements $x,y \in R$ the commutator $[x,y] = xy - yx$. We will frequently use the basic commutator identities: $[xy,z] = x[y,z] + [x,z]y$ and $[x,yz] = y[x,z] + [x,y]z$ for all $x,y \in R$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x,y \in R$. Let α be an automorphism of a ring R. An additive mapping $d : R \to R$ is called an α-derivation if $d(xy) = d(x)\alpha(y) + xd(y)$ holds for all $x,y \in R$. Note that the mapping $d = \alpha - I$ is an α -derivation. Of course, the concept of α -derivation generalizes the concept of derivation, since I-derivation is a derivation. An additive mapping $F : R \to R$ is called a generalized derivation with an associated derivation d of R if $F(xy) = F(x)y + xd(y)$ holds for all $x,y \in R$. Every derivation is a generalized derivation of R. A mapping $f : R \to R$ is called centralizing if $[f(x),x] \in Z(R)$ holds for all $x \in R$, in the special case when $[f(x),x] = 0$ holds for all $x \in R$, the mapping f is said to be commuting on R. Analogously a mapping $f : R \to R$ is called skew-centralizing if $f(x)x + xf(x) \in Z(R)$ and is called skew-commuting if $f(x)x + xf(x) = 0$ holds for all $x \in R$. Posner [9] has proved that the existence of nonzero centralizing derivation on a prime ring forces the ring to be commuttative. Mayne [8] proved that in case there exists a nontrivial centralizing automorphism on a prime ring, then the ring is commuttative.

Bresar [2] has proved that if R is a 2-torsion free semiprime ring and $f : R \to R$ is an additive skew-commuting mapping on R, then $f = 0$. Vukman [13] proved that if there exist a derivation $d : R \to R$ and an automorphism $\alpha : R \to R$, where R is 2-torsion free semiprime ring such that $[d(x)x + x\alpha(x), x] = 0$ holds for all $x \in R$, then d and $\alpha - I$, where I denotes the identity mapping, map R into its center. We extend Vukman results for generalized derivation.

2. MAIN RESULTS

We begin with the following Lemmas which are essential to prove our main results.

Lemma 2.1. [12, Lemma 1] Let R be a semiprime ring. Suppose that the relation $axb + bxc = 0$ holds for all $x \in R$ and some $a,b,c \in R$. In this case, $(a+c)xb = 0$ is satisfied for all $x \in R$.

Lemma 2.2. [14, Lemma 1.3] Let R be a semiprime ring. Suppose that there exists $a \in R$ such that $a[x,y] = 0$ holds for all $x,y \in R$. In this case, $a \in Z(R)$.

Lemma 2.3. [10, Proposition 2.3] Let R be a semiprime ring and let $d : R \to R$ be a commuting α -derivation on R. In this case, d maps R into its center.

Lemma 2.4. [13, Theorem 6] Let R be 2 -torsion free semiprime ring and let $f : R \to R$ be an additive centralizing mappings on R. In this case, f is commuting on R.

Lemma 2.5. [13, Lemma 3] Let R be a semiprime ring and let $f : R \to R$ be an additive mapping. If either $f(x)x = 0$ or $xf(x) = 0$ holds for all $x \in R$, then $f = 0$.

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In [13, Theorem 4] Vukman proved that if R is a semiprime ring, $d : R \to R$ is a derivation of R and α is an automorphism of R such that the mapping $x \to d(x) + \alpha(x)$ is commuting on R, then d, $\alpha - I$ map R into $Z(R)$, the centre of R. We extend the result replacing d by a generalized derivation F of R as follows:

Theorem 2.1 Let R be a semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that the mapping $x \to F(x) + \alpha(x)$ is commuting on R. In this case, d and $\alpha - I$ map R into $Z(R)$.

Proof: The linearization of the relation

$$[F(x) + \alpha(x), x] = 0 \text{ for all } x \in R, \tag{1}$$

gives;

$$[F(x) + \alpha(x), y] + [F(y) + \alpha(y), x] = 0 \text{ for all } x, y \in R, \tag{2}$$

Taking yx instead of y in (2) and using (1), we obtain

$$[F(x) + \alpha(x), y]x + [F(y), x]x + y[d(x), x] + [y, x]d(x) + [\alpha(y), x]\alpha(x) + \alpha(y)[\alpha(x), x] = 0 \text{ for all } x, y \in R. \tag{3}$$

According to relation (2) one can replace in the above relation $[F(x) + \alpha(x), y]x + [F(y), x]x$ by $-[\alpha(y), x]x$ which gives

$$[\alpha(y), x]G(x) + y[d(x), x] + [y, x]d(x) + \alpha(y)[\alpha(x), x] = 0 \text{ for all } x, y \in R, \tag{4}$$

where $G(x)$ denotes $\alpha(x) - x$. Replacing xy for y in (4) we get

$$[\alpha(x), x]\alpha(y)G(x) + \alpha(x)[\alpha(y), x]G(x) + xy[d(x), x] + x[y, x]d(x) + \alpha(x)\alpha(y)[\alpha(x), x] = 0 \text{ for all } x, y \in R. \tag{5}$$

Replacing $\alpha(y)$ by y in the above relation, we obtain

$$[\alpha(x), x]yG(x) + \alpha(x)[y, x]G(x) + xy[d(x), x] + x[y, x]d(x) + \alpha(x)y[\alpha(x), x] = 0 \text{ for all } x, y \in R. \tag{6}$$

Left multiplying (4) by x, replacing $\alpha(y)$ by y and then subtracting from (6), we get

$$[G(x), x]yG(x) + G(x)[y, x]G(x) + G(x)y[G(x), x] = 0 \text{ for all } x, y \in R, \tag{7}$$

where $[\alpha(x), x] = [G(x), x]$, which reduces to

$$xG(x)yG(x) + G(x)y(-G(x)x) = 0 \text{ for all } x, y \in R. \tag{8}$$

Applying Lemma 2.1, the above relation gives

$$[G(x), x]yG(x) = 0 \text{ for all } x, y \in R. \tag{9}$$

Substituting yx for y in (9), we obtain

$$[G(x), x]yxG(x) = 0 \text{ for all } x, y \in R. \tag{10}$$

Right multiplying (9) by x and then subtracting from (10), we get

$$[G(x), x]y[G(x), x] = 0 \text{ for all } x, y \in R. \tag{11}$$

Semiprimeness of R yields that

$$[G(x), x] = 0 \text{ for all } x \in R. \tag{12}$$

We have therefore, $[\alpha(x), x] = 0$, for all $x \in R$, which gives together with the relation (1) yields that

$$[F(x), x] = 0 \text{ for all } x \in R. \tag{13}$$

Linearization of the above relation gives

$$[F(x), y] + [F(y), x] = 0 \text{ for all } x, y \in R. \tag{14}$$

Replacing yx for y in (14)and using (13), we obtain

$$[F(x), y]x + [F(y), x]x + [yd(x), x] = 0 \text{ for all } x, y \in R. \tag{15}$$

Right multiplying (14) by x and then subtracting from (15), we get

$$[yd(x), x] = 0 \text{ for all } x, y \in R. \tag{16}$$

Substituting $d(x)y$ for y in (16) and using (16), we obtain

$$[d(x), x]yd(x) = 0 \text{ for all } x, y \in R. \tag{17}$$

Replacing y by yx in (17), we get

$$[d(x), x]yxd(x) = 0 \text{ for all } x, y \in R. \tag{18}$$

Right multiplying (17) by x and then subtracting from (18), we obtain

$$[d(x), x]y[d(x), x] = 0 \text{ for all } x, y \in R. \tag{19}$$

Semiprimeness of R yields that

$$[d(x), x] = 0 \text{ for all } x \in R. \tag{20}$$

We have therefore proved that G and d are both commuting on R. Now Lemma 2.3 completes the proof of the theorem.

Corollary 2.1. Let R be a 2-torsion free semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that the mapping $x \to F(x) + \alpha(x)$ is centralizing on R. In this case, d and $\alpha - I$ map R into $Z(R)$.

Proof: The proof is an immediate consequence of Lemma 2.4 and Theorem 2.1.

Corollary 2.2 Let R be a noncommutative prime ring of char $R \neq 2$. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that the mapping $x \to F(x) + \alpha(x)$ is centralizing on R. In this case, $d = 0$ and $\alpha = I$.

In [13, Theorem 10] Vukman proved that If R is a semiprime ring, $d : R \to R$ is a derivation of R and α is an automorphism of R such that the mapping $d(x)x + x(\alpha(x) - x) = 0$ for all $x \in R$, then $d = 0$ and $\alpha = I$. We obtain the result in case of a generalized derivation as follows:

Theorem 2.2 Let R be a semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that $F(x)x + x(\alpha(x) - x) = 0$ for all $x \in R$, then $d = 0$ and $\alpha = I$.

Proof By hypothesis, we have

$$F(x)x + xG(x) = 0 \quad \text{for all} \quad x \in R, \tag{21}$$

where $G(x)$ stands for $\alpha(x) - x$. Replacing x by $x + y$ in (21) and using it, we get

$$F(x)y + F(y)x + xG(y) + yG(x) = 0 \quad \text{for all} \quad x, y \in R. \tag{22}$$

Substituting yx for y in (22), we obtain

$$\begin{aligned} &F(x)yx + F(y)x^2 + yd(x)x + xG(y)\alpha(x) + \\ &xyG(x) + yxG(x) = 0 \quad \text{for all} \quad x, y \in R. \end{aligned} \tag{23}$$

Right multiplying (22) by x and then subtracting from (23), we get

$$\begin{aligned} &yd(x)x + xG(y)G(x) + xyG(x) + yxG(x) \\ &-yG(x)x = 0 \quad \text{for all} \quad x, y \in R. \end{aligned} \tag{24}$$

Replacing y by xy in (24), we get

$$\begin{aligned} &xyd(x)x + xG(x)\alpha(y)G(x) + x^2G(y)G(x) + x^2yG(x) + \\ &xyxG(x) - xyG(x)x = 0 \quad \text{for all} \quad x, y \in R. \end{aligned} \tag{25}$$

Left multiplying (24) by x, we obtain

$$\begin{aligned} &xyd(x)x + x^2G(y)G(x) + x^2yG(x) + xyxG(x) \\ &-xyG(x)x = 0 \quad \text{for all} \quad x, y \in R. \end{aligned} \tag{26}$$

Comparing (25) and (26), we get

$$xG(x)\alpha(y)G(x) = 0 \quad \text{for all} \quad x, y \in R. \tag{27}$$

Replacing $\alpha(y)$ by y in (27), we obtain

$$xG(x)yG(x) = 0 \quad \text{for all} \quad x, y \in R. \tag{28}$$

Replacing y by yx in (28), we get

$$xG(x)yxG(x) = 0 \quad \text{for all} \quad x, y \in R. \tag{29}$$

Semiprimeness of R yields that

$$xG(x) = 0 \quad \text{for all} \quad x \in R. \tag{30}$$

Applying Lemma 2.5, we get $G = 0$. Using this relation in (21) we obtain $F(x)x = 0$ for all $x \in R$. Again by Lemma 2.5 we get $F = 0$. This implies that

$$F(x) = 0 \quad \text{for all} \quad x \in R. \tag{31}$$

Replacing x by xy in (31) and using (31), we obtain

$$xd(y) = 0 \quad \text{for all} \quad x, y \in R. \tag{32}$$

In particular $xd(x) = 0$ for all $x \in R$. By Lemma 2.5, we conclude that $d = 0$.

The following theorem is an extension of Theorem 11 of [13].

Theorem 2.3 Let R be a semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that the mapping $x \to F(x)x + x\alpha(x)$ is commuting on R. In this case, d and $\alpha - I$ map R into $Z(R)$.

Proof: We have the relation

$$[F(x)x + x\alpha(x), x] = 0 \quad \text{for all} \quad x \in R. \tag{33}$$

Linearization of (33) yields that

$$\begin{aligned} &[A(x), y] + [F(x)y + F(y)x + x\alpha(y) + \\ &y\alpha(x), x] = 0 \quad \text{for all} \quad x \in R, \end{aligned} \tag{34}$$

where $A(x)$ stands for $F(x)x + x\alpha(x)$. Replacing yx for y in (34), we get

$$\begin{aligned} &[A(x), y]x + [F(x)y + F(y)x, x]x + [yd(x)x, x] + \\ &x[\alpha(y)\alpha(x), x] + [yx\alpha(x), x] = 0 \quad \text{for all} \quad x, y \in R. \end{aligned} \tag{35}$$

According to (34), one can replace in the above relation $[A(x),y]x+[F(x)y+F(y)x,x]x$ by $-[x\alpha(y)+y\alpha(x),x]x$, to obtain

$$x[\alpha(y),x]G(x)-y[\alpha(x),x]x+[y,x][\alpha(x),x]+ y[d(x),x]x+[y,x]d(x)x+x\alpha(y)[\alpha(x),x] +yx[\alpha(x),x]=0 \quad (36)$$

where $G(x)$ denotes $\alpha(x)-x$. Substituting xy for y and y for $\alpha(y)$ in (36), we get

$$x[\alpha(x),x]yG(x)+x\alpha(x)[y,x]G(x)-xy[\alpha(x),x]x +x[y,x][\alpha(x),x]+xy[d(x),x]x+x[y,x]d(x)x+ x\alpha(x)y[\alpha(x),x]=0 \quad (37)$$

Left multiplying (36) by x, we get

$$x^2[\alpha(y),x]G(x)-xy[\alpha(x),x]x+x[y,x][\alpha(x),x]+ xy[d(x),x]x+x[y,x]d(x)x+x^2\alpha(y)[\alpha(x),x]+ xyx[\alpha(x),x]+x[y,x]x\alpha(x)=0 \text{ for all } x,y\in R. \quad (38)$$

Substituting $\alpha(y)$ for y in (38), we have

$$x^2[y,x]G(x)-xy[\alpha(x),x]x+x[y,x][\alpha(x),x]+ xy[d(x),x]x+x[y,x]d(x)x+x^2y[\alpha(x),x]+ xyx[\alpha(x),x]=0 \text{ for all } x,y\in R. \quad (39)$$

Subtracting (37) from (39), we obtain

$$x[G(x),x]yG(x)+xG(x)y[G(x),x]+ xG(x)[y,x]G(x)=0 \text{ for all } x,y\in R, \quad (40)$$

where $[G(x),x]=[\alpha(x),x]$. Collecting terms, the above relation can be written as

$$-x^2G(x)yG(x)+xG(x)yG(x)x=0 \text{ for all } x,y\in R. \quad (41)$$

Substituting yx for y in the above relation, we get

$$-x^2G(x)yxG(x)+xG(x)yxG(x)x=0 \text{ for all } x,y\in R. \quad (42)$$

Applying Lemma 2.1, we get

$$x[G(x),x]yxG(x)=0 \text{ for all } x,y\in R. \quad (43)$$

Putting first in the above relation yx for y, then multiplying the relation (43) from the right side by x, and then subtracting the relations so obtained one from another, we arrive at $x[G(x),x]yx[G(x),x]=0$ for all $x,y\in R$, whence it follows that

$$x[\alpha(x),x]=0 \text{ for all } x\in R. \quad (44)$$

Applying (44) in (33), we get

$$[F(x),x]x=0 \text{ for all } x\in R. \quad (45)$$

Linearizing (44), we obtain

$$x[\alpha(x),y]+x[\alpha(y),x]+y[\alpha(x),x]=0 \text{ for all } x,y\in R. \quad (46)$$

Substituting xy for y in (46), we get

$$x^2[\alpha(x),y]+x\alpha(x)[\alpha(y),x]+xy[\alpha(x),x]=0 \text{ for all } x,y\in R. \quad (47)$$

Left multiplying (46) by x and then subtracting from (47), we obtain

$$xG(x)[\alpha(y),x]=0 \text{ for all } x,y\in R. \quad (48)$$

Substituting y for $\alpha(y)$ in the above relation, we get

$$xG(x)[y,x]=0 \text{ for all } x,y\in R. \quad (49)$$

Replacing y by yz in (49), we arrive at

$$xG(x)y[z,x]=0 \text{ for all } x,y,z\in R. \quad (50)$$

Linearization of x and w in (50), yields that

$$xG(x)y[z,w]+xG(w)y[z,x]+wG(x)y[z,x] =0 \text{ for all } x,y,z,w\in R. \quad (51)$$

Putting in the above relation $[z,w]yxG(x)$ for y and applying the relation (49), we obtain $(xG(x)[z,w]y(xG(x)[z,w])=0$ for all $x,y,z,w\in R$, Semiprimeness of R gives

$$xG(x)[z,w]=0 \text{ for all } x,y,z,w\in R. \quad (52)$$

Applying Lemma 2.5, we obtain

$$G(x)[z,w]=0 \text{ for all } x,z,w\in R. \quad (53)$$

By Lemma 2.2, we conclude that $G(x)\in Z(R)$ for all $x\in R$. In other words, $\alpha-I$ maps R into $Z(R)$. Linearization of (45) gives

$$[F(x),y]x+[F(y),x]x+[F(x),x]y=0 \text{ for all } x,y\in R. \quad (54)$$

Replacing y by yx in (54), we get

$$[F(x),y]x^2+[F(y),x]x^2+[yd(x),x]x+ [F(x),x]yx=0 \text{ for all } x,y\in R. \quad (55)$$

Right multiplying (54) by x and then subtracting from (55), we obtain

$$[yd(x),x]x=0 \text{ for all } x,y\in R. \quad (56)$$

Replacing y by $d(x)y$ in (56) and using (56), we get

$$[d(x),x]yd(x)x = 0 \text{ for all } x,y \in R. \tag{57}$$

Replacing y by xy in (57), we obtain

$$[d(x),x]xyd(x)x = 0 \text{ for all } x,y \in R. \tag{58}$$

Putting first in the above relation yx for y, then multiplying the relation (58) from the right side by x, and then subtracting the relations so obtained one from another, we arrive at $[d(x),x]xy[d(x),x]x = 0$ for all $x,y \in R.$

Semiprimeness of R gives

$$[d(x),x]x = 0 \text{ for all } x \in R. \tag{59}$$

Hence by [11, Theorem 11], d maps R into $Z(R)$.

Theorem 2.4 Let R be a 2-torsion free semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that $[[F(x),x] \pm \alpha(x),x] = 0$ for all $x \in R$. In this case, d and $\alpha - I$ map R into $Z(R)$.

Proof By hypothesis, we have

$$[[F(x),x],x] + [\alpha(x),x] = 0 \text{ for all } x \in R. \tag{60}$$

Linearization of (60) yields that

$$[[F(y),x],x] + [[F(x),y],x] + [[F(x),x],y] + [\alpha(y),x] + [\alpha(x),y] = 0 \text{ for all } x,y \in R, \tag{61}$$

Replacing y by x in (61) and using (60), we get

$$[[F(x),x],x] = 0 \text{ for all } x \in R. \tag{62}$$

By [5, Theorem 3.4], d maps R into $Z(R)$. Using (62) in (60), we obtain

$$[\alpha(x),x] = 0 \text{ for all } x \in R. \tag{63}$$

Now Lemma 2.3 completes the proof of the theorem.Similarly we can prove the case $[[F(x),x] - \alpha(x),x] = 0$ for all $x \in R$.

Theorem 2.5 Let R be a 2-torsion free semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that $[[F(x) \pm [\alpha(x),x],x] = 0$ for all $x \in R$. In this case, R is commutative and d maps R into $Z(R)$.

Proof: By hypothesis, we have

$$[F(x),x] + [[\alpha(x),x],x] = 0 \text{ for all } x \in R. \tag{64}$$

Linearization of (64) yields that

$$[F(y),x] + [F(x),y] + [[\alpha(y),x],x] + [[\alpha(x),y],x] + [[\alpha(x),x],y] = 0 \text{ for all } x,y \in R, \tag{65}$$

Replacing y by x in (65) and using (64), we get

$$[[\alpha(x),x],x] = 0 \text{ for all } x \in R. \tag{66}$$

This implies that

$$[x,[x,\alpha(x)]] = 0 \text{ for all } x \in R. \tag{67}$$

Replacing $\alpha(y)$ by y in (67) to get

$$[x,[x,y]] = 0 \text{ for all } x,y \in R. \tag{68}$$

This implies that $[x,y] \in Z(R)$ for all $x,y \in R$. Therefore we can write

$$[[x,y],r] = 0 \text{ for all } x,y,r \in R. \tag{69}$$

Substituting yx for y in (69) and using (69), we get

$$[x,y][x,r] = 0 \text{ for all } x,y,r \in R. \tag{70}$$

Replacing r by ry in the above relation and using it, we obtain

$$[x,y]r[x,y] = 0 \text{ for all } x,y,r \in R. \tag{71}$$

Semiprimeness of R yields that

$$[x,y] = 0 \text{ for all } x,y \in R. \tag{72}$$

This implies that R is commutative. Putting (66) in (64) to get

$$[F(x),x] = 0 \text{ for all } x \in R. \tag{73}$$

By Theorem 2.1 we obtain d maps R into $Z(R)$.Similarly we can prove the case $[F(x) - [\alpha(x),x],x] = 0$ for all $x \in R$.

Theorem 2.6 Let R be a 2-torsion free semiprime ring. Suppose that $F : R \to R$ is a generalized derivation with an associated derivation $d : R \to R$ and $\alpha : R \to R$ is an automorphism such that $[[F(x) \pm \alpha(x),x],x] = 0$ for all $x \in R$. In this case, d maps R into $Z(R)$ and R is commutative.

Proof By hypothesis, we have

$$[[F(x),x],x] + [[\alpha(x),x],x] = 0 \text{ for all } x \in R. \tag{74}$$

Linearization of (74) yields that

$$[[F(y),x],x]+[[F(x),y],x]+[[F(x),x],y]+[[\alpha(y),x],x] \\ +[[\alpha(x),y],x]+[[\alpha(x),x],y]=0 \text{ for all } x,y\in R. \quad (75)$$

Substituting x for y and y for $\alpha(y)$ respectively in (75) and using (74), we get

$$[[F(x),x],x]=0 \text{ for all } x\in R. \quad (76)$$

By Theorem 2.4, we obtain d maps R into $Z(R)$. Applying the above relation in (74), we obtain

$$[[\alpha(x),x],x]=0 \text{ for all } x\in R. \quad (77)$$

By Theorem 2.5 we get the required result. Similarly we can prove the case $[[F(x)-\alpha(x),x],x]=0$ for all $x\in R$.

The following example illustrates that the above Theorems do not hold for arbitrary rings and torsion condition in the hypothesis is not superfluous.

Example: Consider $R=\left\{\begin{pmatrix}0&a&b\\0&0&c\\0&0&0\end{pmatrix} \mid a,b,c\in\mathbb{Z}_2\right\}$.

R is neither a semiprime ring nor 2-torsion free. Define maps $F,d,\alpha : R\to R$ by

$F\begin{pmatrix}0&a&b\\0&0&c\\0&0&0\end{pmatrix}=\begin{pmatrix}0&0&b\\0&0&0\\0&0&0\end{pmatrix}$, $d\begin{pmatrix}0&a&b\\0&0&c\\0&0&0\end{pmatrix}=\begin{pmatrix}0&0&b\\0&0&c\\0&0&0\end{pmatrix}$, and $\alpha\begin{pmatrix}0&a&b\\0&0&c\\0&0&0\end{pmatrix}=\begin{pmatrix}0&-a&-b\\0&0&c\\0&0&0\end{pmatrix}$. It can be verified that F is a generalized derivation with an associated derivation d and $d=\alpha-I$ is an α -derivation of R satisfying the hypothesis of Theorem 2.1 - Theorem 2.6. But R is not commuttative.

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