Mode-I Crack Problem in Generalized Thermo-microstretch with Harmonic Wave under Three Theories

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Abstract: A general model of the equations of generalized thermo-microstretch for an infinite space weakened by a finite linear opening Mode-I crack are solving. The material is homogeneous isotropic elastic half space. The crack is subjected to prescribed temperature and stress distribution. The formulation is applied to generalized thermoelasticity theories, using mathematical analysis with the purview of the Lord-Şhulman (LS involving one relaxation time) and Green-Lindsay (GL includes two relaxation times) theories with respect to the classical dynamical coupled theory (CD). The harmonic wave method has been used to getting the exact expression of Normal displacement, Normal stress force, couple stresses, microstress and temperature distribution. The variations of the considered fields with the horizontal distance are explained graphically. A comparison also is made between the three theories and for different depths.

Keywords: Mode-I Crack, L-S theory, GL theory, thermoelasticity, microrotation, microstretch.

1. INTRODUCTION

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g. concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the clastomer and polymer type, e.g. polymethyl- methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves i.e. for the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant, this influence of microstructure results in the development of new type of waves are not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, concrete are typical media with microstructures. More generally, most of the natural and manmade materials including engineering, geological and biological media possess a micro-structure. Eringen and Şuhubi [1] and Eringen [2] developed the linear theory of micropolar elasticity. Othman [3] studied the relaxation effects on thermal shock problems in elastic half space of generalized magneto-thermoelastic waves under three theories. Othman [4] construct a model of the two-dimensional equations of generalized magneto-thermoelasticity with two relaxation times in an isotropic elastic medium with the modulus of elasticity being dependent on the reference temperature. Eringen [5] introduced the theory of microstretch elastic solids. This theory is a generalization of the theory of micropolar elasticity [2,6] and a special case of the micromorphic theory. The material points of microstretch elastic solids can stretch and contract independently of their translations and rotations. The microstretch continua is used to characterize composite materials and various porous media [7]. The basic results in the theory of micro stretch elastic solids were obtained in the literature [8-11].

The theory of thermomicrostretch elastic solids was introduced by Eringen [7]. In the frame-work of the theory of thermomicrostretch solids Eringen established a uniqueness theorem for the mixed initial-boundary value problem. The theory was illustrated through the solution of one dimensional waves and compared with lattice dynamical results. The asymptotic behavior of solutions and an existence result were presented by Bofill and Quintanilla [12]. A reciprocal theorem and a representation of Galerkin type were presented by De Cicco and Nappa [13]. De Cicco and Nappa [14] extended a linear theory of thermomicrostretch elastic solids that permits the transmission of heat as thermal waves at finite speed. The theory is based on the entropy production inequality proposed by Green and Laws [15]. In [14], the uniqueness of the solution of the mixed initial-boundary-value problem is also investigated. The basic results and an extensive review on the theory of thermo-microstretch elastic solids can be found in the book of Eringen [8]. The coupled theory of thermoelasticity has been extended by including the thermal relaxanon time in the constitutive equations by Lord and Shulman [16] and Green and Lindsay [17].

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E-ISSN: 2409-5761/16

These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity.

Othman and Lotfy [18] studied two-dimensional problem of generalized magneto-thermoelasticity under the effect of temperature dependent properties. Othman and Lotfy [19] studied the effect of magnetic field and rotation of the 2-D problem of a fiber-reinforced thermoelastic under three theories with influence of gravity. Othman and Lotfy [20] studied the plane waves in generalizedthermo-microstretch elastic half-space by using a general model of the equations of generalized thermo-microstretch for a homogeneous isotropic elastic half space. Othman and Lotfy [21] studied the generalized thermo-microstretch elastic medium with temperature dependent properties for different theories. Othman and Lotfy [22] studied the effect of magnetic field and inclined load in micropolar thermoelastic medium possessing cubic symmetry under three theories. The normal mode analysis was used to obtain the exact expression for the temperature distribution, thermal stresses, and the displacement components.

In the recent years, considerable efforts have been devoted the study of failure and cracks in solids. This is due to the application of the latter generally in industry and particularly in the fabrication of electronic components. Most of the studies of dynamical crack problem are done using the equations of coupled or even uncoupled theories of thermoelasticity [23-31]. This is suitable for most situations where long time effects are sought. However, when short time are important, as in many practical situations, the full system of generalized thermoelastic equations must be used [16].

The purpose of the present paper is to obtain the normal displacement, temperature, normal force stress, and tangential couple stress in a microstretch elastic solid. The normal mode analysis used for the problem of generalized thermo-microstretch for an infinite space weakened by a finite linear opening Mode-I crack is solving for the considered variables. The distributions of the considered variables are represented graphically. A comparison is carried out among the temperature, stresses and displacements as calculated from the generalized thermoelasticity (L-S), (G-L) and (CD) theories for the propagation of waves in semi-infinite microstretch elastic solids.

2. FORMULATION OF THE PROBLEM

Following Eringen [3], Green and Lindsay [15] and Lord and Şhulman [16], the constitutive equations and field equations for a linear isotropic generalized thermo-microstretch elastic solid in the absence of body forces are obtained. We consider Cartesian coordinate system (x, y, z) having origin on the surface $y = 0$ and z-axis pointing vertically into the medium, the region G given by $G = \{(x, y, z) \mid, -\infty < x < \infty, -\infty < z < \infty\}$ with a crack on the x-axis, $|x| \le a$, is considered. The crack surface is subjected to a known temperature and normal stresses distributions. There are many types of crack and this study will be devoted to Mode-I shown in Figure **1**.

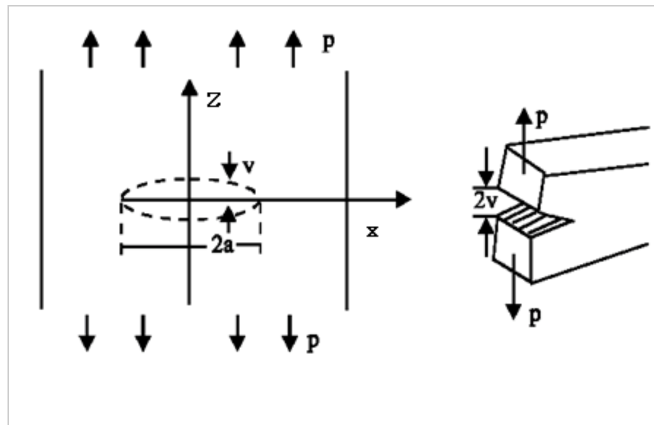

Figure 1: Displacement of an external Mode-I crack.

The fundamental system of field equations consists of the equations of motion for a linear, isotropic generalized thermo-microstretch elastic soiled medium are given by

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+(\mu+k)\nabla^2\vec{u}+k(\nabla\times\vec{\varphi})\lambda_0\nabla\varphi^* -\hat{\gamma}\left(1+v_0\frac{\partial}{\partial t}\right)\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2} \quad (1)$$

$$(n+n+n)\nabla(\nabla.\vec{\varphi})-n\nabla\times(\nabla\times\vec{\varphi}) +k(\nabla\times\vec{u})-2k\vec{\varphi}=jn\frac{\partial^2\vec{\varphi}}{\partial t^2} \quad (2)$$

$$n_0\nabla^2\varphi^*-\frac{1}{3}n_1\varphi^*-\frac{1}{3}n_0(\nabla.\vec{u})+\frac{1}{3}r\varphi_1 (1+n_0\frac{\partial}{\partial t})T=\frac{3}{2}nj\frac{\partial^2\varphi^*}{\partial t^2} \quad (3)$$

The equation of heat conduction;

$$K\nabla^2 T = nC_E(n_1 + n_0\frac{\partial}{\partial t})\dot{T} + \\ r\varphi T_0(n_1 + n_0 n_0\frac{\partial}{\partial t})\dot{e} + r\varphi_1 T_0\frac{\partial \varphi^*}{\partial t} \quad (4)$$

The constitutive law for the theory of generalized thermoelasticity with two relaxation times;

$$n_{il} = (n_0\varphi^* + n u_{r,r})n_{il} + (n+k)u_{l,i} + \\ n u_{i,l} - k n_{ilr} n_r - r\varphi(1 + n_0\frac{\partial}{\partial t})T n_{il} \quad (5)$$

The field equations and constitutive relations for thermo-microstretch generalized thermoelastic medium

$$m_{il} = \acute{a}\,\varphi_{r,r}\ddot{a}_{il} + \hat{a}\ddot{o}_{i,l} + \tilde{a}\,\varphi_{l,i} \quad (6)$$

$$\square^{\square}{}_i = n_0\,\varphi_i^* \quad (7)$$

The relation between strain-displacement:

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad (8)$$

The state of plane strain parallel to the xz -plane is defined by;

$$u_1 = u(x,z,t),\ u_2 = 0,\ u_3 = w(x,z,t),\ \varphi_1 = \varphi_3 = 0,$$

$$\varphi_2 = \varphi_2(x,z,t),\ \varphi^* = \varphi^*(x,z,t), \quad (9)$$

The field equations (1)-(4) reduce to;

$$(n+n)(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x\partial z}) + (n+k)(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}) - \\ k\frac{\partial \phi_2}{\partial z} + \lambda_0\frac{\partial \phi^*}{\partial x} - r\varphi\left(1 + n_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \quad (10)$$

$$(\lambda+\mu)(\frac{\partial^2 u}{\partial x\partial z} + \frac{\partial^2 w}{\partial z^2}) + (\mu+k)(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}) \\ +k\frac{\partial \phi_2}{\partial x} + \lambda_0\frac{\partial \varphi^*}{\partial z} - \hat{\gamma}(1 + n_0\frac{\partial}{\partial t})\frac{\partial T}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}, \quad (11)$$

$$\gamma(\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2}) - 2k\phi_2 + k(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) = j\rho\frac{\partial^2 \phi_2}{\partial t^2} \quad (12)$$

$$\alpha_0(\frac{\partial^2 \varphi^*}{\partial x^2} + \frac{\partial^2 \varphi^*}{\partial z^2}) - \frac{1}{3}\lambda_1\varphi^* - \frac{1}{3}\lambda_0(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) + \\ \frac{1}{3}\hat{\gamma}_1(1 + \nu_0\frac{\partial}{\partial t})T = \frac{3}{2}\rho j\frac{\partial^2 \varphi^*}{\partial t^2} \quad (13)$$

$$K(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}) = \rho C_E(n_1 + \tau_0\frac{\partial}{\partial t})\frac{\partial T}{\partial t} \\ + \hat{\gamma}T_0(n_1 + n_0\tau_0\frac{\partial}{\partial t})\frac{\partial e}{\partial t} + \hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t} \quad (14)$$

where;

$\hat{a} = (3\ddot{e}+2\grave{i}+k)\acute{a}_{t_1}$, $\hat{a}_1 = (3\ddot{e}+2\grave{i}+k)\acute{a}_{t_2}$ and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (15)$$

The constants $\hat{a}$ and $\hat{a}_1$ depend on mechanical as well as the thermal properties of the body and the dot denote the partial derivative with respect to time.

Equations (10)-(14) are the field equations of the generalized thermo-microstretch elastic solid, applicable to the (L-S) theory, the (G-L) theory, as well as the classical coupled theory (CD), as follows:

1. The equations of the coupled thermo-microstretch (CD) theory, when

$$n_0 = 0,\quad n_1 = 1,\quad \tau_0 = \nu_0 = 0 \quad (16)$$

Eqs. (10), (11), (13) and (14) has the form;

$$\rho\ddot{u} = (\lambda+\mu)(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x\partial z}) + (\mu+k)(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}) \\ -k\frac{\partial \phi_2}{\partial z} + \lambda_0\frac{\partial \varphi^*}{\partial x} - \hat{\gamma}\frac{\partial T}{\partial x}, \quad (17)$$

$$\rho\ddot{w} = (\lambda+\mu)(\frac{\partial^2 u}{\partial x\partial z} + \frac{\partial^2 w}{\partial z^2}) + (\mu+k)(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}) \\ +k\frac{\partial \phi_2}{\partial x} + \lambda_0\frac{\partial \varphi^*}{\partial z} - \hat{\gamma}\frac{\partial T}{\partial z} \quad (18)$$

$$c_3^2(\frac{\partial^2 \varphi^*}{\partial x^2} + \frac{\partial^2 \varphi^*}{\partial z^2}) - c_4^2\varphi^* - c_5^2(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) + c_6^2 T = \frac{\partial^2 \varphi^*}{\partial t^2}, \quad (19)$$

$$K(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}) = \rho C_E\frac{\partial T}{\partial t} + \hat{\gamma}T_0\frac{\partial e}{\partial t} + \hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t}. \quad (20)$$

The constitutive relation can be written as;

$$\acute{o}_{xx} = \ddot{e}_0\varphi^* + (\ddot{e} + 2\grave{i} + k)\frac{\partial u}{\partial x} + \ddot{e}\frac{\partial w}{\partial z} - \hat{a}T, \quad (21)$$

$$\acute{o}_{zz} = \ddot{e}_0\varphi^* + (\ddot{e} + 2\grave{i} + k)\frac{\partial w}{\partial z} + \ddot{e}\frac{\partial u}{\partial x} - \hat{a}T, \quad (22)$$

$$\sigma_{xz} = \mu \frac{\partial u}{\partial z} + (\mu + k)\frac{\partial w}{\partial x} + k\varphi_2, \quad (23)$$

$$\sigma_{xz} = \mu \frac{\partial w}{\partial x} + (\mu + k)\frac{\partial u}{\partial z} + k\varphi_2, \quad (24)$$

$$m_{xy} = \gamma \frac{\partial \varphi_2}{\partial x}, \quad (25)$$

$$m_{zy} = \tilde{a} \frac{\partial \varphi_2}{\partial z}. \quad (26)$$

where;

$$c_3^2 = \frac{2\alpha_0}{3\rho j}, c_4^2 = \frac{2\lambda_1}{9\rho j}, c_5^2 = \frac{2\lambda_0}{9\rho j}, c_6^2 = \frac{2\hat{\gamma}_1}{9\rho j}. \quad (27)$$

2. Lord-Shulman (L-S) theory, when;

$$n_1 = n_0 = 1, \quad \nu_0 = 0, \quad \pi_0 > 0 \quad (28)$$

Eqs (10), (11) and (13) are the same as Eqs (17), (18) and (19) and Eq. (14) has the form;

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \left(\frac{\partial}{\partial t} + \pi_0 \frac{\partial^2}{\partial t^2}\right) \{\rho C_E T + \hat{\gamma} T_0 e\} + \hat{\gamma}_1 T_0 \frac{\partial \phi^*}{\partial t} \quad (29)$$

3. Green-Lindsay (G-L) theory, when;

$$n_1 = 1, \quad n_0 = 0 \quad , \quad \nu_0 \geq \tau_0 > 0 \quad (30)$$

Eqs (10), (11) and (13) remain unchanged and Eq. (14) has the form;

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_E \left(1 + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \hat{\gamma} T_0 \frac{\partial e}{\partial t} + \hat{\gamma}_1 T_0 \frac{\partial \varphi^*}{\partial t} \quad (31)$$

4. The corresponding equations for the generalized micropolar thermo-elasticity without stretch can be obtained from the above mentioned cases by taking:

$$\alpha_0 = \lambda_0 = \lambda_1 = \phi^* = 0$$

For convenience, the following non-dimensional variables are used:

$$\bar{x}_i = \frac{\omega^*}{C_2} x_i, \; \bar{u}_i = \frac{\rho C_2 \omega^*}{\hat{\gamma} T_0} u_i, \; \bar{t} = \omega^* t,$$
$$\bar{\tau}_0 = \omega^* \tau_0, \bar{\nu}_0 = \omega^* \nu_0, \bar{T} = \frac{T}{T_0}, \; \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\hat{\gamma} T_0},$$
$$\bar{m}_{ij} = \frac{\omega^*}{C_2 \hat{\gamma} T_0} m_{ij}, \; \bar{\varphi}_2 = \frac{\rho C_2^2}{\hat{\gamma} T_0} \varphi_2,$$
$$\bar{\lambda}_3 = \frac{\omega^*}{C_2 \hat{\gamma} T_0} \lambda_3, \; \bar{\varphi}^* = \frac{\rho C_2^2}{\hat{\gamma} T_0} \varphi^*, \; \omega^* = \frac{\rho C_E C_2^2}{K}, \; C_2^2 = \frac{\mu}{\rho}. \quad (32)$$

Using Eqs. (32), Eqs. (10)-(14) become (dropping the dashed for convenience);

$$\frac{\partial^2 u}{\partial t^2} = \frac{(\mu + k)}{\rho C_2^2} \nabla^2 u + \frac{(\mu + \lambda)}{\rho C_2^2} \frac{\partial e}{\partial x} - \frac{k}{\rho C_2^2} \frac{\partial \phi_2}{\partial z} + \frac{\lambda_0}{\rho C_2^2} \frac{\partial \varphi^*}{\partial x} - \left(1 + \nu_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x} \quad (33)$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{(\mu + k)}{\rho C_2^2} \nabla^2 w + \frac{(\mu + \lambda)}{\rho C_2^2} \frac{\partial e}{\partial z} + \frac{k}{\rho C_2^2} \frac{\partial \phi_2}{\partial x} + \frac{\lambda_0}{\rho C_2^2} \frac{\partial \varphi^*}{\partial z} - \left(1 + \nu_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial z} \quad (34)$$

$$\frac{j\rho C_2^2}{\gamma} \frac{\partial^2 \phi_2}{\partial t^2} = \nabla^2 \phi_2 - \frac{2kC_2^2}{\gamma w^*} \phi_2 + \frac{kC_2^2}{\gamma w^{*2}} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \quad (35)$$

$$\left(\frac{c_3^2}{C_2^2} \nabla^2 - \frac{c_4^2}{w^{*2}} - \frac{\partial^2}{\partial t^2}\right)\varphi^* - \frac{c_5^2}{w^{*2}} e + a_9 \left(1 + \nu_0 \frac{\partial}{\partial t}\right) T = 0 \quad (36)$$

$$\nabla^2 T - \left(n_1 + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} - \frac{\hat{\gamma}^2 T_0}{\rho K w^*}\left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} = \frac{\hat{\gamma}\hat{\gamma}_1 T_0}{\rho K w^*} \frac{\partial \varphi^*}{\partial t}. \quad (37)$$

Assuming the scalar potential functions $\varphi(x,z,t)$ and $\psi(x,z,t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial z}, \; w = \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial x}. \quad (38)$$

Using (38) in Eqs. (33)-(37), we obtain.

$$\left[\nabla^2 - a_0 \frac{\partial^2}{\partial t^2}\right]\varphi - a_0 \left(1 + \nu_0 \frac{\partial}{\partial t}\right) T + a_1 \varphi^* = 0 \quad (39)$$

$$\left[\nabla^2 - a_2 \frac{\partial^2}{\partial t^2}\right]\psi - a_3 \phi_2 = 0 \quad (40)$$

$$\left[\nabla^2 - 2a_4 - a_5 \frac{\partial^2}{\partial t^2}\right]\phi_2 - a_4 \nabla^2 \psi = 0 \quad (41)$$

$$\left[a_6 \nabla^2 - a_7 - \frac{\partial^2}{\partial t^2}\right]\varphi^* - a_8 \nabla^2 \varphi + a_9 \left(1 + \nu_0 \frac{\partial}{\partial t}\right) T = 0 \quad (42)$$

$$\left[\nabla^2-(n_1\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\right]T-\varepsilon$$
$$(n_1\frac{\partial}{\partial t}+n_0\tau_0\frac{\partial^2}{\partial t^2})\nabla^2\varphi-\varepsilon_1\frac{\partial\varphi^*}{\partial t}=0 \quad (43)$$

where;

$$c_1^2=\frac{\lambda+2\mu+k}{\rho},\quad a_0=\frac{C_2{}^2}{C_1{}^2},\quad a_1=\frac{\lambda_0}{\lambda+2\mu+k},\quad a_2=\frac{\rho C_2{}^2}{\mu+k},$$
$$a_3=\frac{k}{\mu+k},\quad a_4=\frac{kC_2{}^2}{\gamma w^{*2}},\quad a_5=\frac{\rho jC_2{}^2}{\gamma},\quad a_6=\frac{C_3{}^2}{C_2{}^2},$$
$$a_7=\frac{C_4{}^2}{w^{*2}},\quad a_8=\frac{C_5{}^2}{w^{*2}},\quad a_9=\frac{2\hat{\gamma}_1C_2{}^2}{9\hat{\gamma}\, jw^{*2}},\quad \varepsilon=\frac{\hat{\gamma}^2T_0}{\rho\omega^*K},$$
$$\varepsilon_1=\frac{\hat{\gamma}\hat{\gamma}_1T_0}{\rho\omega^*K}. \quad (44)$$

The solution of the considered physical variables can be decomposed in terms of normal mode as the following form:

$$[\varphi,\psi,\varphi^*,\varphi_2,\sigma_{il},m_{il},T](x,z,t)=$$
$$[\bar{\varphi}(x),\bar{\psi}(x),\bar{\varphi}^*(x),\bar{\varphi}_2(x),\bar{\sigma}_{il}(x),\bar{m}_{il}(x),\bar{T}(x)]\exp(\omega t+iaz). \quad (45)$$

where $[\bar{\varphi},\bar{\psi},\bar{\varphi}^*,\bar{\varphi}_2,\bar{\sigma}_{il},\bar{m}_{il},\bar{T}](x)$ are the amplitude of the functions ù is a complex and a is the wave number in the z-direction and,

Using Eq. (45), then Eqs. (39)–(43) become respectively;

$$(D^2-A_1)\bar{\varphi}-A_2\bar{T}+a_1\bar{\varphi}^*=0, \quad (46)$$

$$(D^2-A_3)\bar{\psi}-a_3\bar{\varphi}_2=0 \quad (47)$$

$$(D^2-A_4)\bar{\varphi}_2-a_4(D^2-a^2)\bar{\psi}=0, \quad (48)$$

$$(a_6D^2-A_5)\bar{\varphi}^*-a_8(D^2-a^2)\bar{\varphi}+A_8\bar{T}=0, \quad (49)$$

$$\left[(D^2-a^2)-A_6\right]\bar{T}-A_7(D^2-a^2)\bar{\varphi}-\varepsilon_1\omega\bar{\varphi}^*=0, \quad (50)$$

where $D=\frac{d}{dx}$,

$$A_1=a^2+a_0\omega^2, \quad (51)$$

$$A_2=a_0\left(1+\nu_0\omega\right), \quad (52)$$

$$A_3=a^2+a_2\omega^2, \quad (53)$$

$$A_4=a^2+2a_4+a_5\omega^2, \quad (54)$$

$$A_5=a^2a_6+a_7+\omega^2, \quad (55)$$

$$A_6=\omega\left(n_1+\tau_0\omega\right), \quad (56)$$

$$A_7=\varepsilon\omega\left(n_1+n_0\tau_0\omega\right). \quad (57)$$

$$A_8=a_9\left(1+\nu_0\omega\right), \quad (58)$$

Eliminating $\bar{\varphi}_2,\bar{\psi}$ between Eqs. (47) and (48), we get the following fourth order ordinary differential equation satisfied by $\bar{\varphi}_2$ and $\bar{\psi}$;

$$\left[D^4-AD^2+B\right]\left\{\bar{\varphi}_2(x),\bar{\psi}(x)\right\}=0. \quad (59)$$

Eliminating $\bar{\varphi}$, $\bar{T}$ and $\bar{\varphi}^*$ between Eqs. (46), (49) and (50) we obtain the following sixth order ordinary differential equation satisfied by $\bar{\varphi}^*(x),\bar{\varphi}(x)$ and $\bar{T}(x)$;

$$\left[D^6-CD^4+ED^2-H\right]\left\{\bar{\varphi}^*(x),\bar{\varphi}(x),\bar{T}(x)\right\}=0. \quad (60)$$

Where;

$$A=A_3+A_4+a_3a_4, \quad (61)$$

$$B=A_3A_4+a^2a_3a_4, \quad (62)$$

$$C=\frac{g_3(g_7+g_8)-g_5}{g_3}, \quad (63)$$

$$E=\frac{a^2g_3g_8+\varepsilon_1\omega g_1A_2-g_6-g_5g_7-g_4g_8}{g_3}, \quad (64)$$

$$H=\frac{a^2g_4g_8+g_6g_7-\varepsilon_1\omega g_2A_2}{g_3}, \quad (65)$$

$$g_1=A_8-a_8A_2,\quad g_2=a^2a_8A_2-A_1A_8,\quad g_3=a_6A_2,$$
$$g_4=a_1A_8+A_5A_2,\quad g_5=g_4-A_1g_3-a_1g_1,$$
$$g_6=A_1g_4+a_1g_2,\quad g_7=a^2A_6+A_7,\quad g_8=A_2A_8. \quad (66)$$

The solution of Eqs. (59) and (60), has the form

$$\bar{\psi}(x) = \sum_{j=1}^{2} M_j(a,\omega)\ e^{-k_j x}, \tag{67}$$

$$\bar{\varphi}_2(x) = \sum_{j=1}^{2} M'_j(a,\omega)\ e^{-k_j x}, \tag{68}$$

$$\bar{\varphi}(x) = \sum_{n=3}^{5} M_n(a,\omega)\ e^{-k_n x}, \tag{69}$$

$$\bar{\varphi}^*(x) = \sum_{n=3}^{5} M'_n(a,\omega)\ e^{-k_n x}, \tag{70}$$

$$\bar{T}(x) = \sum_{n=3}^{5} M''_n(a,\omega)\ e^{-k_n x}, \tag{71}$$

where $M_j(a,\omega)$, $M'_j(a,\omega)$, $M_n(a,\omega)$, $M'_n(a,\omega)$ and $M''_n(a,\omega)$ are some parameters depending on a and $\omega . k_j^2, (j=1,2)$ are the roots of the characteristic equation of Eq. (59) and $k_n^2, (n=3,4,5)$ are the roots of the characteristic equation of Eq. (60).

Using Eqs. (67)- (71) into Eqs. (46) and (50) we get the following relations

$$\bar{\varphi}_2(x) = \sum_{j=1}^{2} a_j^* M_j(a,\omega)\ e^{-k_j x}, \tag{72}$$

$$\bar{\varphi}^*(x) = \sum_{n=3}^{5} b_n^* M_n(a,\omega)\ e^{-k_n x}, \tag{73}$$

$$\bar{T}(x) = \sum_{n=3}^{5} c_n^* M_n(a,\omega)\ e^{-k_n x}. \tag{74}$$

where,

$$a_j^* = \frac{(k_j^2 - A_3)}{a_3}, \qquad j = 1,2, \tag{75}$$

$$b_n^* = \frac{g_1 k_n^2 + g_2}{g_3 k_n^2 + g_4}, \qquad n = 3,4,5, \tag{76}$$

$$c_n^* = \frac{g_3 k_n^4 + g_5 k_n^2 - g_6}{A_2 (g_3 k_n^2 + g_4)}, \qquad n = 3,4,5. \tag{77}$$

4. APPLICATION

The plane boundary subjects to an instantaneous normal point force and the boundary surface is isothermal, the boundary conditions at the vertical plan $y=0$ and in the beginning of the crack at $x=0$ are $\sigma_{zz} = -p(x), \ |x| < a \quad T = f(x), \quad |x| < a$ and $\frac{\partial T}{\partial z} = 0 \quad |x| > a$.

$$\sigma_{xz} = 0, \qquad -\infty < x < \infty, \tag{78}$$

$$m_{xy} = 0, \qquad -\infty < x < \infty,$$

$$\lambda_z = 0 \qquad -\infty < x < \infty,$$

Using (32), (38), (39)-(43) with the non-dimensional boundary conditions and using (67), (69), (72)-(74), we obtain the expressions of displacement components, force stress, coupled stress and temperature distribution for microstretch generalized thermoelastic medium as follows:

$$\bar{u}(x) = ia(M_1 e^{-k_1 x} + M_2 e^{-k_2 x}) \\ -k_3 M_3 e^{-k_3 x} - k_4 M_4 e^{-k_4 x} - k_5 M_5 e^{-k_5 x}, \tag{79}$$

$$\bar{w}(x) = k_1 M_1 e^{-k_1 x} + k_2 M_2 e^{-k_2 x} \\ + ia(M_3 e^{-k_3 x} + k_4 M_4 e^{-k_4 x} + k_5 M_5 e^{-k_5 x}), \tag{80}$$

$$\bar{w}_{zz}(x) = s_1 M_1 e^{-k_1 x} + s_2 M_2 e^{-k_2 x} + \\ s_3 M_3 e^{-k_3 x} + s_4 M_4 e^{-k_4 x} + s_5 M_5 e^{-k_5 x}, \tag{81}$$

$$\bar{\sigma}_{xz}(x) = r_1 M_1 e^{-k_1 x} + r_2 M_2 e^{-k_2 x} \\ + r_3 M_3 e^{-k_3 x} + r_4 M_4 e^{-k_4 x} + r_5 M_5 e^{-k_5 x}, \tag{82}$$

$$\bar{m}_{xy}(x) = q_1 M_1 e^{-k_1 x} + q_2 M_2 e^{-k_2 x}, \tag{83}$$

$$\bar{T}(x) = c_3^* M_3 e^{-k_3 x} + c_4^* M_4 e^{-k_4 x} + c_5^* M_5 e^{-k_5 x}, \tag{84}$$

$$\lambda_z = f_8 (b_3^* M_3 e^{-k_3 x} + b_4^* M_4 e^{-k_4 x} + b_5^* M_5 e^{-k_5 x}). \tag{85}$$

where;

$s_1 = iak_1(f_2 - f_3)$, $\qquad s_2 = iak_2(f_2 - f_3)$,

$s_3 = f_1 b_3^* - a^2 f_2 + f_3 k_3^2 - c_3^*(1 + \nu_0 \omega)$,

$s_4 = f_1 b_4^* - a^2 f_2 + f_3 k_4^2 - c_4^*(1 + \nu_0 \omega)$,

$s_5 = f_1 b_5^* - a^2 f_2 + f_3 k_5^2 - c_5^*(1 + \nu_0 \omega)$,

$r_1 = a_1^* f_6 - a^2 f_4 - f_5 k_1^2$, $r_2 = a_2^* f_6 - a^2 f_4 - f_5 k_2^2$,

$r_3 = -iak_3(f_4 + f_5)$, $r_4 = -iak_4(f_4 + f_5)$, $r_5 = iak_5(f_4 + f_5)$,

$q_1 = -f_7 a_1^* k_1$, $q_2 = -f_7 a_2^* k_2$, $f_1 = \frac{\lambda_0}{\rho c_2^2}$, $f_2 = \frac{\lambda + 2\mu + k}{\rho c_2^2}$,

$f_3 = \frac{\lambda}{\rho c_2^2}$, $f_4 = \frac{\mu}{\rho c_2^2}$, $f_5 = \frac{\mu + k}{\rho c_2^2}$, $f_6 = \frac{k}{\rho c_2^2}$, $f_7 = \frac{\gamma \omega^{*2}}{\rho c_2^4}$ and

$$f_8 = \frac{\alpha_0 \omega^{*2}}{\rho c_2^4}. \tag{86}$$

Applying the boundary conditions (78) at the surface $x = 0$ of the plate, we obtain a system of five equations. After applying the inverse of matrix method, we obtain the values of the five constants $M_j, j = 1,2$, and $M_n, n = 3,4,5$. Hence, we obtain the expressions of displacements, force stress, couple stress and temperature distribution for microstretch generalized thermoelastic medium.

5. NUMERICAL RESULTS AND DISCUTIONS

In order to illustrate our theoretical results obtained in preceding section and to compare these in the context of various theories of thermoelasticity, we now present some numerical results. In the calculation process, we take the case of copper crystal as material subjected to mechanical and thermal disturbances for numerical calculations consider the material medium as that of copper. Since, ù is the complex constant then we taken $\omega = \omega_0 + i\xi$. The other constants of the problem are taken as $\omega_0 = -2; \xi = 1$ and a=1.

The results are shown in Figures **2–15**. The graph shows the three curves predicted by different theories of thermoelasticity. In these figures, the solid lines represent the solution in the Coupled theory, the dotted lines represent the solution in the generalized Lord and Shulman theory and dashed lines represent the solution derived using the Green and Lindsay theory. We notice that the results for the temperature, the displacement and stresss distribution when the relaxation time is including in the heat equation are distinctly different from those when the relaxation time is not mentioned in heat equation, because the thermal waves in the Fourier's theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non-Fourier case. This demonstrates clearly the difference between the coupled and the generalized theories of thermoelasticity.

For the value of z, namely z = 0.1, were substituted in performing the computation. It should be noted (Figure **2**) that in this problem, the crack's size, x is taken to be the length in this problem so that $0 \le x \le 3$, $z = 0$ represents the plane of the crack that is symmetric with respect to the z-plane. It is clear from the graph that T has maximum value at the beginning of the crack ($x = 0$), it begins to fall just near the crack edge ($x = 3$), where it experiences sharp decreases (with maximum negative gradient at the crack's end). The value of temperature quantity converges to zero with increasing the distance x.

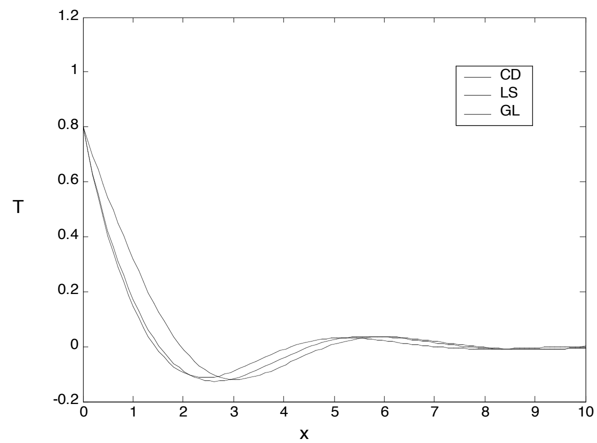

Figure 2: Variation of temperature distribution T with different theories.

Figure **3**, the horizontal displacement, u, begins with decrease then smooth increases again to reach its maximum magnitude just at the crack end. Beyond it u falls again to try to retain zero at infinity. Figure **4**, the vertical displacement w, we see that the displacement component w always starts from the zero value and terminates at the zero value. Also, at the crack end to reach minimum value, beyond reaching zero at the double of the crack size (state of particles equilibrium).

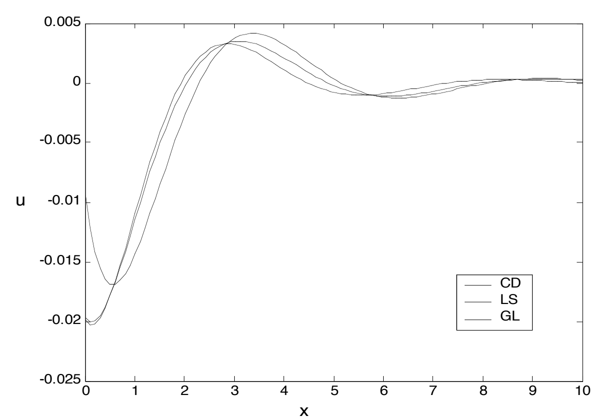

Figure 3: Variation of displacement distribution u with different theories.

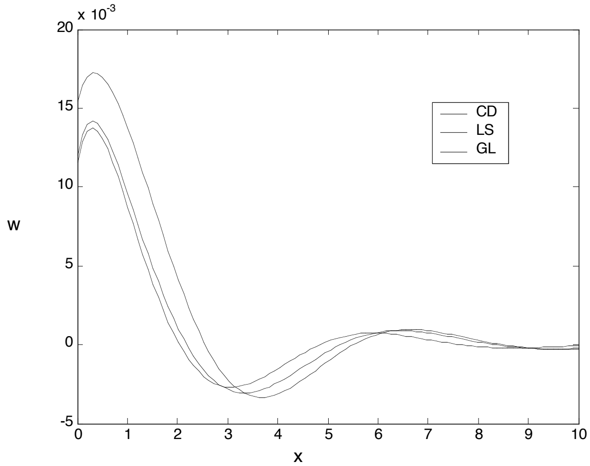

Figure 4: Variation of displacement distribution w with different theories.

The displacements u and w show different behaviours, because of the elasticity of the solid tends to resist vertical displacements in the problem under investigation. Both of the components show different behaviours, the former tends to increase to maximum just before the end of the crack. Then it falls to a minimum with a highly negative gradient. Afterwards it rises again to a maximum beyond about the crack end.

The stresse component, σ_{zz} reach coincidence with negative value (Figure **5**) and satisfy the boundary condition at $x=0$, reach the maximum value near the end of crack ($x \approx 3$) and converges to zero with increasing the distance x. Figure **6**, shows that the stress component σ_{xz} satisfy the boundary condition at $x=0$ and had a different behaviour. It decreases in the start and start decreases (maximum) in the context of the three theories until reaching the crack end. These trends obey elastic and thermoelastic properties of the solid under investigation.

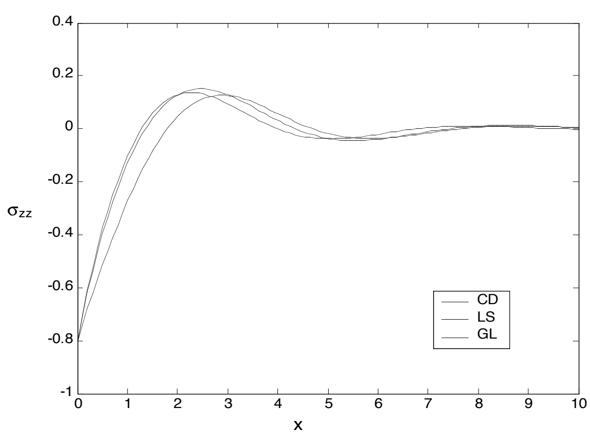

Figure 5: Variation of stress distribution σ_{zz} with different theories.

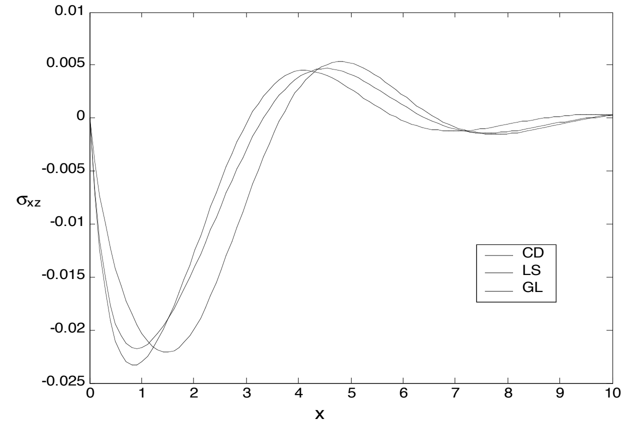

Figure 6: Variation of stress distribution σ_{xz} with different theories.

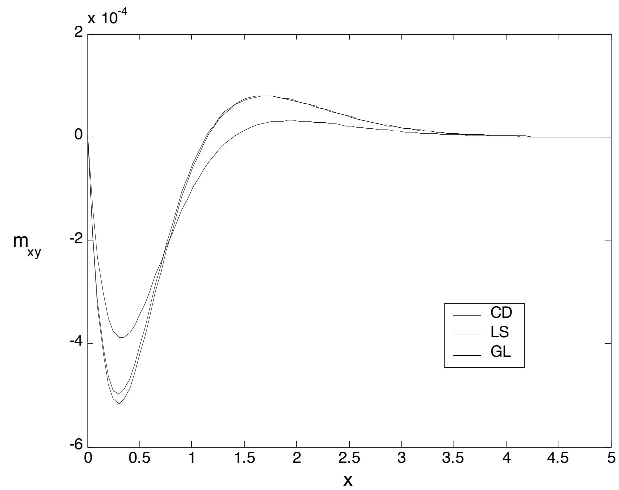

Figure 7: Variation of tangential couple stress m_{xy} with different theories.

Figure **7**, the tangential coupled stress m_{xy} satisfies the boundary condition at $x=0$. It decreases in the start and start increases (maximum) in the context of the three theories until reaching the crack end. The values of microstress for λ_z satisfy the boundary condition at $x=0$, begins with increase then decreases again to reach its minimum magnitude just near the crack end, beyond reaching zero at the double of the crack size (state of particles equilibrium), as depicted in Figure **8**.

Figure **9-15** show the comparison between the temperature T, displacement components u, W, the force stresses components σ_{zz}, σ_{xz}, the tangential coupled stress m_{xy} and the microstress λ_z, the case of different three values of z, (namely z= 0.1, z=0.2 and z=0.3) under GL theory. It should be noted (Figure9) that in this problem. It is clear from the graph that T

has maximum value at the beginning of the crack ($x=0$), it begins to fall just near the crack edge ($x=3$), where it experiences sharp decreases (with maximum negative gradient at the crack's end). Graph lines for both values of y show different slopes at crack ends according to y-values. In other words, the temperature line for z = 0.1 has the highest gradient when compared with that of z = 0.2 and z = 0.3 at the first of the range. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach the reference temperature of the solid. These results obey physical reality for the behaviour of copper as a polycrystalline solid.

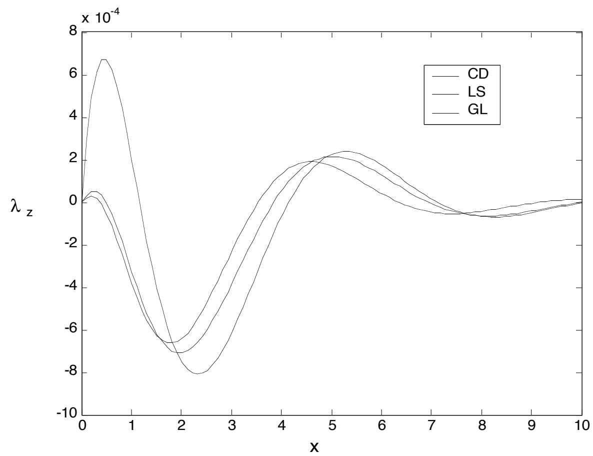

Figure 8: Variation of microstress $\ddot{e}_Z$ with different theories.

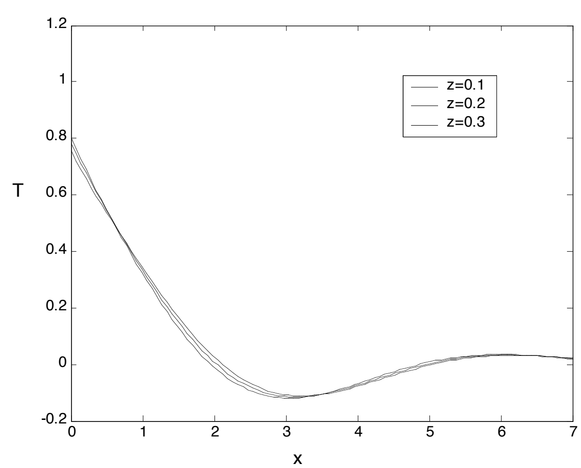

Figure 9: Variation of temperature distribution T for different vertical distances, under GL theory.

Figure **10**, the horizontal displacement u, despite the peaks (for different vertical distances z=0.1, z=0.2 and z=0.3) occur at equal value of x, the magnitude of the maximum displacement peak strongly depends on the vertical distance y. it is also clear that the rate of change of u increases with increasing y as we go farther apart from the crack. On the other hand, Figure **11** shows atonable increase of the vertical displacement, W, near the crack end to reach minimum value beyond $x=3$ reaching zero at the double of the crack size(state of particles equilibrium).

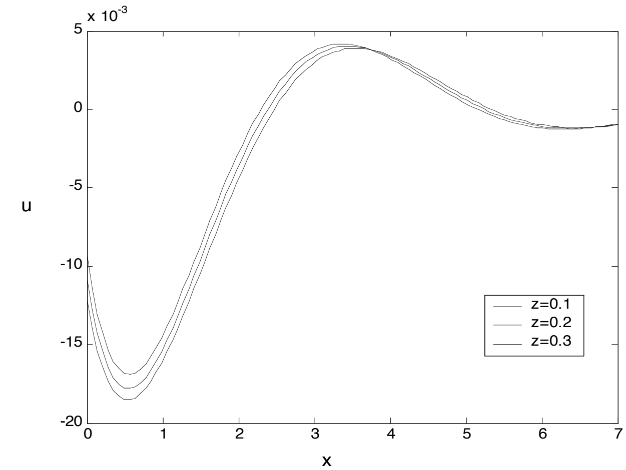

Figure 10: Variation of displacement distribution u for different vertical distances, under GL theory.

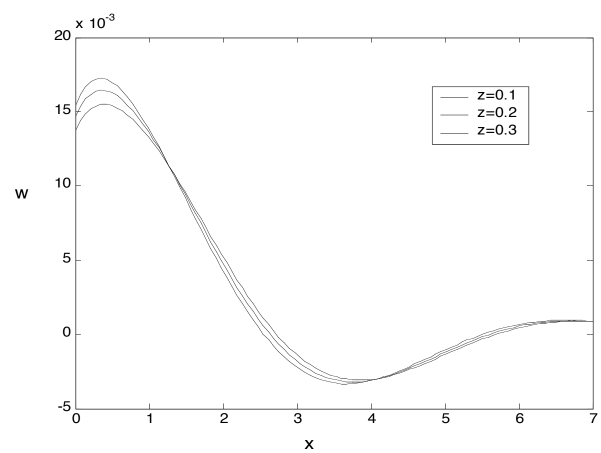

Figure 11: Variation of displacement distribution w for different vertical distances, under GL theory.

Figure **12**, the vertical stresses σ_{zz} Graph lines for both values of z show different slopes at crack ends according to z-values. In other words, the σ_{zz} component line for z = 0.1 has the highest gradient when compared with that of z = 0.2 and z = 0.3 at the edge of the crack. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach zero after their relaxations at infinity. Variation of y has a serious effect on both magnitudes of mechanical stresses. These trends obey elastic and thermoelastic properties of the solid under investigation. Figure **13**, shows that the stress component σ_{xz} satisfy the boundary condition, the line for z = 0.3 has the highest gradient when compared with that of z = 0.2 and z= 0.1 in the range $0 \le x \le 2.5$, the line for z = 0.1 has the highest gradient when compared with that of z = 0.2 and z= 0.3 in the range $2.5 \le x \le 5$ and converge to zero when $x > 5$. These trends obey elastic and thermoelastic properties of the solid.

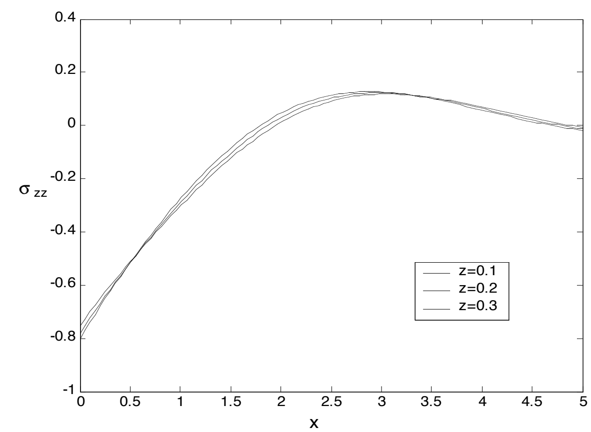

Figure 12: Variation of stress distribution O'_{zz} for different vertical distances, under GL theory.

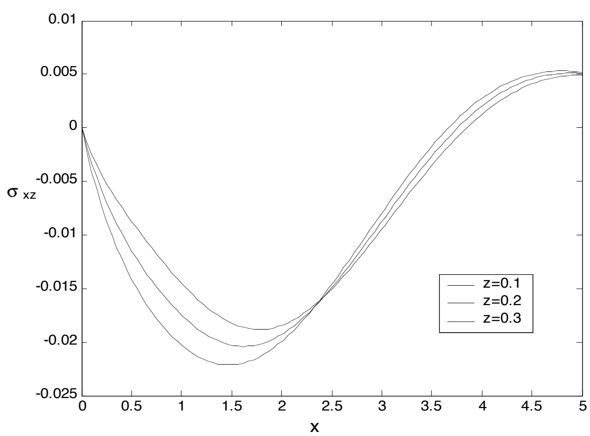

Figure 13: Variation of stress distribution O_{xz} for different vertical distances, under GL theory.

Figure **14**, the tangential coupled stress m_{xy} it decreases in the start and start increases (maximum) in the context of the three values of z until reaching the crack end, for z = 0.3 has the highest gradient when compared with that of z = 0.2 and z= 0.1 at the edge of the crack. All lines begin to coincide when the horizontal distance x is beyond the edge of the crack. Figure **15**, shown the values of microstress for λ_z it increases in the start and start decreases (minimum) in the context of the three values of z until reaching nearly the crack end, for z = 0.3 has the highest gradient when compared with that of z = 0.2 and z= 0.1 at the edge of the crack. All lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach zero after their relaxations at infinity.

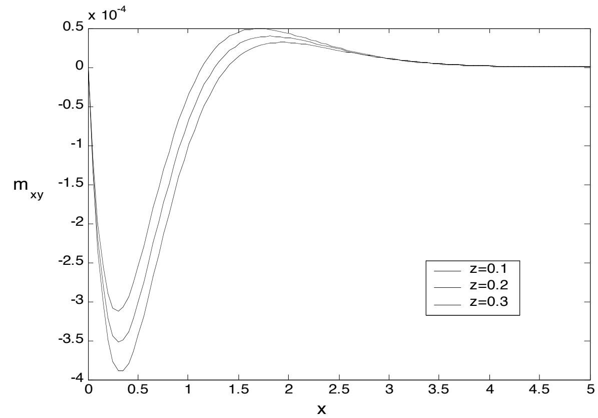

Figure 14: Variation of tangential couple stress m_{xy} for different vertical distances, under GL theory.

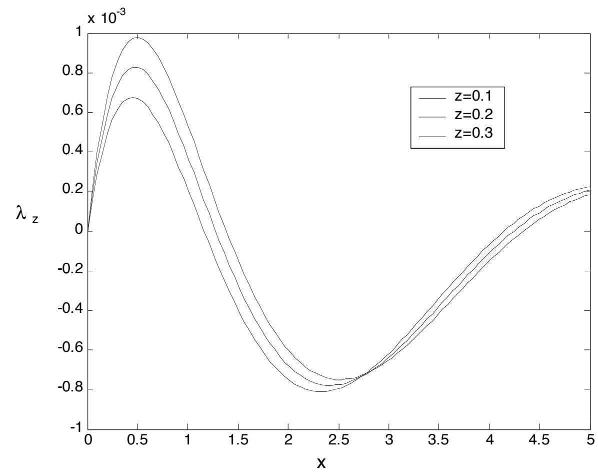

Figure 15: Variation of microstress $\ddot{e}_z$ for different vertical distances, under GL theory.

CONCLUSIONS

The models of generalized thermo-microstretch for an infinite space weakened by a finite linear opening Mode-I crack is solving. The physical quantities are given analytically and illustrated graphically by Normal mode method. The effects of the thermal relaxation time (three theories), the case of different three values of the depth are discussed. The following conclusions can be made:

1. The curves in the context of the (CD), (L-S) and (G-L) theories decrease exponentially with increasing x, this indicate that the thermoelastic waves are unattenuated and nondispersive,

where purely thermoelastic waves undergo both attenuation and dispersion.

2. The presence of microstretch plays a significant role in all the physical quantities.

3. The curves of the physical quantities with (L-S) theory in most of figures are lower in comparison with those under (G-L) theory, due to the relaxation times.

4. Analytical solutions based upon normal mode analysis for themoelastic problem in solids have been developed and utilized.

5. A linear opening mode-I crack has been investigated and studied for copper solid.

6. Temperature, radial and axial distributions were estimated at different distances from the crack edge.

7. The stresses distributions, the tangential coupled stress and the values of microstress were evaluated as functions of the distance from the crack edge.

8. Crack dimensions are significant to elucidate the mechanical structure of the solid.

9. Cracks are stationary and external stress is demanded to propagate such cracks.

10. It can be concluded that a change of volume is attended by a change of the temperature while the effect of the deformation upon the temperature distribution is the subject of the theory of thermoelasticity.

11. The value of all the physical quantities converges to zero with an increase in distance y and All functions are continuous.

NOMENCLATURE

λ, μ Lame's constants ($\lambda = 7.76 \times 10^{10}\ N/m^2$, $\mu = 3.86 \times 10^{10}\ N/m^2$).

ρ Density ($\rho = 8954\ kgm^{-1}$).

C_E Specific heat at constant strain ($C_E = 383.1\ J\, kg^{-1} K^{-1}.$)

ν Poisson's ratio.

t Time.

τ_0, ν_0 Relaxation times.

T Absolute temperature.

σ_{ij} Components of stress tensor.

e_{ij} Components of strain tensor.

u_i Components of displacement vector.

K Thermal conductivity ($K = 0.6 \times 10^{-2}\ cal/cm\ \sec$).

J Current density vector.

φ Rotation vector.

T_0 Reference temperature chosen so that $\left|\frac{T - T_0}{T_0}\right| < 1$ ($T_0 = 293K$).

φ^* The scalar microstretch.

m_{ij} Couple stress tensor.

λ_i^* First moment tensor.

δ_{ij} Kroneker delta.

ε_{ijr} The alternate tensor.

e Dilatation.

α_t Coefficients of linear thermal expansions ($\alpha_t = 1.78 \times 10^{-5}\ K^{-1}$).

k, α, β, γ Micropolar constants.

$\alpha_0, \lambda_0, \lambda_1$ Microstretch elastic constants.

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Received on 08-03-2016 Accepted on 08-06-2016 Published on 14-07-2016

DOI: http://dx.doi.org/10.15377/2409-5761.2016.03.01.6