Uni-Type Modal Operators On Intuitionistic Fuzzy Sets

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Abstract: Intuitionistic Fuzzy Modal Operator was defined by Atanassov, he introduced the generalization of these modal operators. After this study, some authors defined some modal operators which are called one type and two type modal operator on intuitionistic fuzzy sets. In these studies, some extensions and characteristic properties were obtained.

In this paper we defined new operators and examine some properties of them. In view of conclusions, it is shown that these operators are both one type and two type modal operators on Intuitionistic Fuzzy Sets. So, these common type modal operators are called uni-type modal operators on Intuitionistic Fuzzy Sets.

Keywords: Diagram of modal operators, Intuitionistic fuzzy operators, uni-type modal operators.

1. INTRODUCTION

The theory of fuzzy sets (FSs) was first stated by Zadeh, [12], in 1965. Let X be a set then the function $\mu : X \to [0,1]$ is called a fuzzy set over X and it is shown by $\mu \in FS(X)$. For $x \in X$, $\mu(x)$ is called the membership degree of x on A and the nonmembership degree is $1-\mu(x)$.

Atanassov [1] defined intuitionistic fuzzy sets (IFS) in 1983. While the nonmembership degree for each element of the universe is fixed in fuzzy set theory. In intuitionistic fuzzy set theory, nonmembership degree is a more or less independent degree; satisfying the condition that it is smaller than 1- *membership degree*. So, if X is a universe then there exist membersip and nonmembership degrees for each $x \in X$, respectively $\mu(x)$ and $\nu(x)$ such that $0 \le \mu(x) + \nu(x) \le 1$.

IFS A is determined with the membership and non-membership of $\mu_A(x) \in FS(X)$, $\nu_A(x) \in FS(X)$ for $x \in X$, resp. Although the sum of the degrees of membership and not being a member of an element in FS theory is 1. But, in IFS theory, this sum is less than 1. Besides this, if $A \in IFS(X)$ then μ, $\nu \in FS(X)$ and $1-\mu \le \nu$ and $1-\nu \le \mu$.

An IFS A is said to be contained in an IFS B notation $A \subseteq B$) if and only if for all $x \in X$: $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$. It is clear that $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

The intersection and the union of two IFSs A and B on X is defined as following;

$$A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$$

$$A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$$

Definition 1.1. [2] Let $A \in IFS$ and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$. The set

$$A^c = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$$

is called the complement of A.

The notion of Modal Operators on IFSs was firstly introduced by Atanassov [2].

Definition 1.2. [2] Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$.

1) $\square A = \{< x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2} >: x \in X\}$

2) $\Diamond A = \{< x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2} >: x \in X\}$

After this definition, in 2001, Atanassov, in [3], defined the extension of these operators as following,

Definition 1.3. [3] Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$, $\alpha \in [0,1]$.

1) $\lceil_{\alpha} A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha >: x \in X\}$

2) $\lfloor_{\alpha} A = \{< x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x) >: x \in X\}$

In these operators $\lceil_{\alpha}$ and $\lfloor_{\alpha}$ If we choose $\alpha = \frac{1}{2}$, we get the operators $\lceil$and $\lfloor$, resp. Therefore, the operators $\lceil_{\alpha}$ and $\lfloor_{\alpha}$ are the extensions of the operators $\lceil$and $\lfloor$resp. Some relationships between these operators were studied by several authors [9, 11]

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In 2004, the second extension of these operators was introduced by Dencheva in [9].

Definition 1.4. [9] Let X be a set, $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\}\in IFS(X)$ and $\alpha,\beta\in[0,1]$. The sets $\lceil_{\alpha,\beta}A$ and $\lfloor_{\alpha,\beta}A$ are defined as follows:

1) $\lceil_{\alpha,\beta}A=\{<x,\alpha\mu_A(x),\alpha\nu_A(x)+\beta>: x\in X\}$ where $\alpha+\beta\in[0,1]$.

2) $\lfloor_{\alpha,\beta}A=\{<x,\alpha\mu_A(x)+\beta,\alpha\nu_A(x)>: x\in X\}$ where $\alpha+\beta\in[0,1]$.

The concepts of the modal operators are being introduced and studied by different researchers, [3-6], [9, 10, 11], etc.

In 2006, the third extension of the above operators was studied by Atanassov. He defined the following operators in [4]

Definition 1.5. [4] Let X be a set, $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\}\in IFS(X)$ and $\alpha,\beta,\gamma\in[0,1]$, $\max\{\alpha,\beta\}+\gamma\le 1$. The sets $\lfloor_{\alpha,\beta,\gamma}(A)$ and $\lfloor_{\alpha,\beta,\gamma}(A)$ are defined as follows:

1) $\lfloor_{\alpha,\beta,\gamma}(A)=\{<x,\alpha\mu_A(x),\beta\nu_A(x)+\gamma>: x\in X\}$

2) $\lfloor_{\alpha,\beta,\gamma}(A)=\{<x,\alpha\mu_A(x)+\gamma,\beta\nu_A(x)>: x\in X\}$

If we choose $\alpha=\beta$ and $\gamma=\beta$ in above operators then we can see easily that $\lceil_{\alpha,\alpha,\gamma}=\lceil_{\alpha,\beta}$ and $\lfloor_{\alpha,\alpha,\gamma}=\lfloor_{\alpha,\beta}$. Therefore, we can say that $\lceil_{\alpha,\beta,\gamma}$ and $\lfloor_{\alpha,\beta,\gamma}$ are the extensions of the operators

In 2007, the author [7] defined a new operator and studied some of its properties. This operator is named $E_{\alpha,\beta}$ and defined as follows:

Definition 1.6. [7] Let X be a set and $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\}\in IFS(X)$, $\alpha,\beta\in[0,1]$. We define the following operator:

$$E_{\alpha,\beta}(A)=\{<x,\beta(\alpha\mu_A(x)+1-\alpha),\alpha(\beta\nu_A(x)+1-\beta)>: x\in X\}$$

If we choose $\alpha=1$ and write α insteed of β we get the operator $\lceil_\alpha$. Similarly, if $\beta=1$ is chosen and writen insteed of β, we get the operator $\lfloor_\alpha$.

These extensions have been investigated by several authors [10], [5,6]. In particular, the authors have made significant contributions to these operators.

In 2007, Atanassov introduced the operator $|_{\alpha,\beta,\gamma,\delta}$ which is a natural extension of all these operators in [5].

Definition 1.7. [5] Let X be a set, $A\in IFS(X)$, $\alpha,\beta,\gamma,\delta\in[0,1]$ such that $\max(\alpha,\beta)+\gamma+\delta\le 1$. The operator $|_{\alpha,\beta,\gamma,\delta}$ defined by

$$|_{\alpha,\beta,\gamma,\delta}=\{<x,\alpha\mu_A(x)+\gamma,\beta\nu_A(x)+\delta>: x\in X\}$$

In 2008, Atanassov defined this most general operator $\lfloor_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}$ as following:

Definition1.8. [6] Let X be a set, $A\in IFS(X)$, $\alpha,\beta,\gamma,\delta,\varepsilon,\zeta\in[0,1]$ such that $\max(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\le 1$ and $\min(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\ge 0$. Then the operator $\lfloor_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}$ defined by

$$\lfloor_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}(A)=\{<x,\alpha\mu_A(x)-\varepsilon\nu_A(x)+\gamma,\beta\nu_A(x)-\zeta\mu_A(x)+\delta>: x\in X\}$$

In 2010, the author [8] defined a new operator as follows:

Definition 1.9. [8] Let X be a set and $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\}\in IFS(X)$, $\alpha,\beta,\omega\in[0,1]$. We define the following operator:

$$Z^{\omega}_{\alpha,\beta}(A)=\{<x,\beta(\alpha\mu_A(x)+\omega-\omega.\alpha),\alpha(\beta\nu_A(x)+\omega-\omega.\beta)>: x\in X\}$$

In 2013 author defined the following operator which is a generalization of $Z^{\omega}_{\alpha,\beta}$.

Definition 1.10. [8] Let X be a set and $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\}\in IFS(X)$, $\alpha,\beta,\omega,\theta\in[0,1]$. We define the following operator:

$$Z^{\omega,\theta}_{\alpha,\beta}(A)=\{<x,\beta(\alpha\mu_A(x)+\omega-\omega.\alpha),\alpha(\beta\nu_A(x)+\theta-\theta.\beta)>: x\in X\}$$

The operator $Z^{\omega,\theta}_{\alpha,\beta}$ is a generalization of $Z^{\omega}_{\alpha,\beta}$, and also, $E_{\alpha,\beta}$, $\lceil_{\alpha,\beta}$ and $\lfloor_{\alpha,\beta}$

Before defining new operators which are generalization of both one type and second type modal operators, we will recall definitions of second type modal operators.

Definition 1.11. [2] Let X be universal and $A\in IFS(X)$, $\alpha\in[0,1]$. The set $D_\alpha(A)$ defined as follows:

$$D_\alpha(A)=\{<x,\mu_A(x)+\alpha\pi_A(x),\nu_A(x)+(1-\alpha)\pi_A(x)>: x\in X\}$$

Definition 1.12. [2] Let X be universal and $A \in IFS(X)$, $\alpha,\beta \in [0,1]$ and $\alpha+\beta \leq 1$. The set $F_{\alpha,\beta}(A)$ defined as follows:

$$F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) >: x \in X\}$$

Definition 1.13. [2] Let X be universal and $A \in IFS(X)$, $\alpha,\beta \in [0,1]$. The set $G_{\alpha,\beta}(A)$ defined as follows:

$$G_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) >: x \in X\}$$

Definition 1.14. [2] Let X be universal and $A \in IFS(X)$, $\alpha,\beta \in [0,1]$. The following sets are defined;

1) $H_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \nu_A(x) + \beta\pi_A(x) >: x \in X\}$

2) $H^*_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \nu_A(x) + \beta(1 - \alpha\mu_A(x) - \nu_A(x)) >: x \in X\}$

3) $J_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \beta\nu_A(x) >: x \in X\}$

4) $J^*_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha(1 - \mu_A(x) - \beta\nu_A(x)), \beta\nu_A(x) >: x \in X\}$

5) $\square\ (A) = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$

6) $\Diamond\ (A) = \{< x, 1 - \nu_A(x), \nu_A(x) >: x \in X\}$

After these studies the diagram of all modal operator a is given as following;

2. NEW OPERATORS $\lceil^{\omega}_{\alpha,\beta}$, $\lfloor^{\omega}_{\alpha,\beta}$, $E^{\omega,\theta}_{\alpha,\beta}$, $B_{\alpha\beta}$ AND $\{_{\alpha\beta}$

Definition 2.1. Let X be a universal, $A \in IFS(X)$ and. We define the following operators: $\alpha,\beta,\omega \in [0,1]$.

1) $\lceil^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha(\beta\nu_A(x) + \omega - \omega\beta)\rangle : x \in X\}$

2) $\lfloor^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + \omega - \omega\alpha), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$

It is clear that; $\lceil^{\omega}_{\alpha,\beta}$, $\lfloor^{\omega}_{\alpha,\beta}$ are IF operators.

From this definition, we get the following new diagram which is the extesion of the last diagram of intuitionistic fuzzy operators on IFSs in Figure **2**.

Now we present some fundamental properties and relationships of new operators.

Theorem 2.1. Let X be a universal, $A \in IFS(X)$ and $\alpha,\beta,\omega \in [0,1]$.

1) If $\beta \leq \alpha$ then $\lceil^{\omega}_{\alpha,\beta}(\lceil^{\omega}_{\beta,\alpha}(A)) \subseteq \lceil^{\omega}_{\beta,\alpha}(\lceil^{\omega}_{\alpha,\beta}(A))$

2) If $\beta \leq \alpha$ then $\lfloor^{\omega}_{\alpha,\beta}(\lfloor^{\omega}_{\beta,\alpha}(A)) \subseteq \lfloor^{\omega}_{\beta,\alpha}(\lfloor^{\omega}_{\alpha,\beta}(A))$

Proof (1) If we use $\beta \leq \alpha$ then we get,

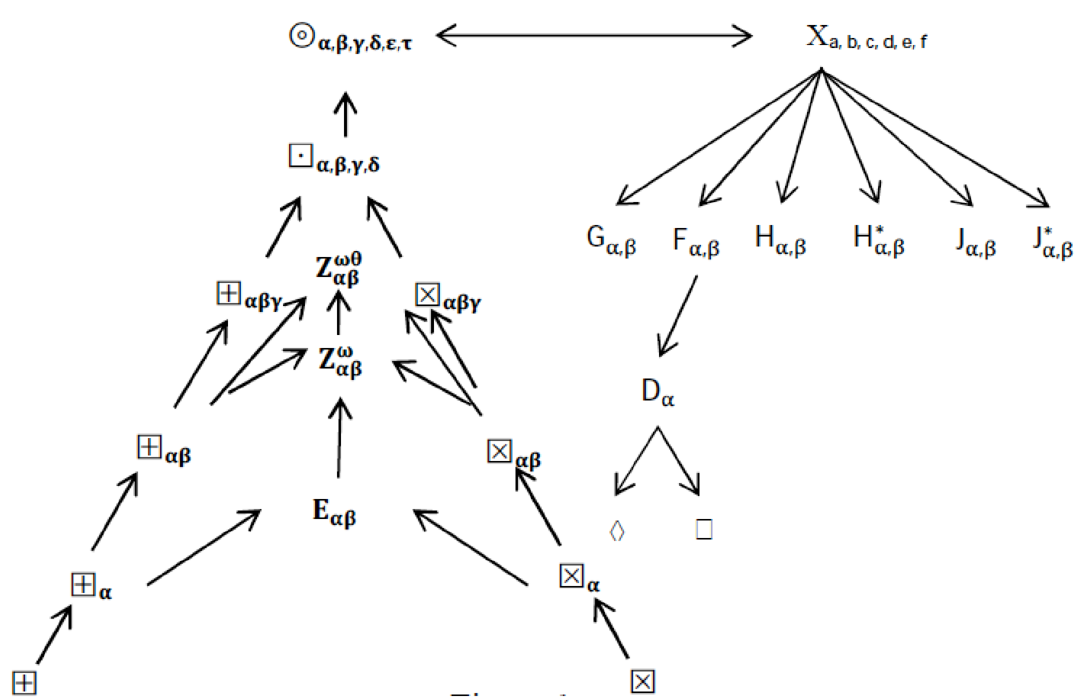

Figure 1:

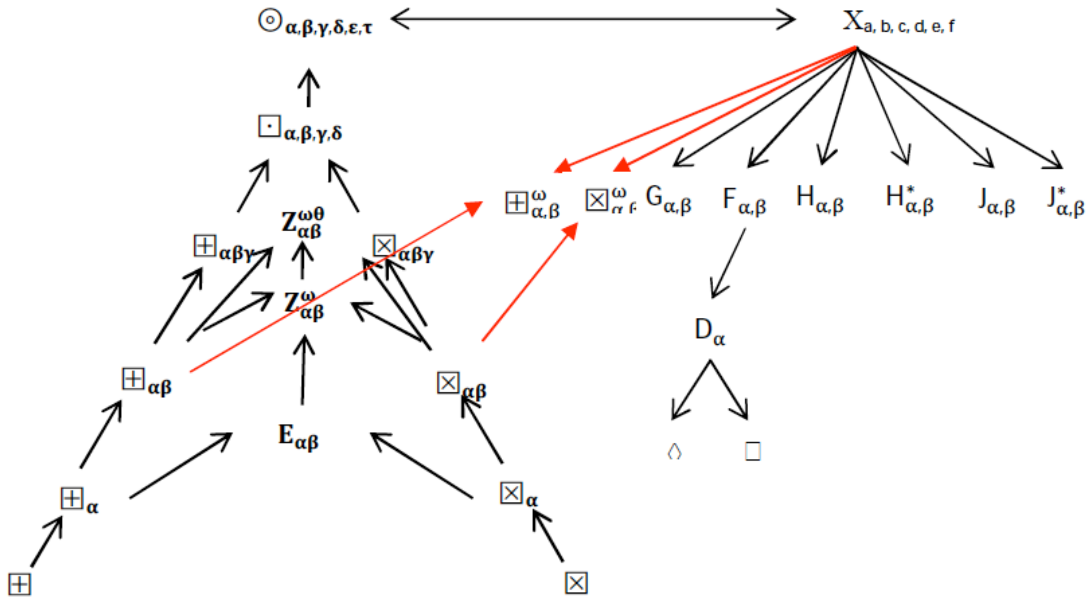

Figure 2:

$$\beta \le \alpha \Rightarrow (\beta-\alpha)(\beta+\alpha+2\alpha\beta) \le 0$$
$$\Rightarrow \beta^2(1+2\alpha) \le \alpha^2(1+2\beta)$$
$$\Rightarrow \beta^2(1+2\alpha)\omega \le \alpha^2(1+2\beta)\omega$$

and with this inequality we can say

$$\alpha\beta\mu_A(x)+\alpha\beta(1-\beta)\nu_A(x)+\beta(1-\alpha)(\alpha\beta\nu_A(x)+\beta\omega-\alpha\beta\omega)$$
$$\le \alpha\beta\mu_A(x)+\alpha\beta(1-\alpha)\nu_A(x)+\alpha(1-\beta)(\alpha\beta\nu_A(x)+\alpha\omega-\alpha\beta\omega)$$

On the other hand

$$\beta \le \alpha \Rightarrow (\beta-\alpha)(\alpha\beta-1) \ge 0$$
$$\Rightarrow \alpha\beta^2+\alpha-\alpha\beta \ge \alpha^2\beta+\beta-\alpha\beta$$
$$\Rightarrow \alpha\beta^2\omega+\alpha\omega-\alpha\beta\omega \ge \alpha^2\beta\omega+\beta\omega-\alpha\beta\omega$$

with this we can say

$$\alpha^2\beta^2\nu_A(x)+\alpha\beta^2\omega-\alpha^2\beta^2\omega+\alpha\omega-\alpha\beta\omega \ge$$
$$\alpha^2\beta^2\nu_A(x)+\alpha^2\beta\omega-\alpha^2\beta^2\omega+\beta\omega-\alpha\beta\omega$$

So we get

$$\lceil^{\omega}_{\alpha,\beta}(\lceil^{\omega}_{\beta,\alpha}(A)) \subseteq \lceil^{\omega}_{\beta,\alpha}(\lceil^{\omega}_{\alpha,\beta}(A))$$

We can show the property (2) with the same way.

Proposition 2.1. Let X be a universal, $A \in IFS(X)$ and $\alpha,\beta \in [0,1)$. Then the following statements hold:

1) $\lceil^{\frac{\beta}{1-\alpha}}_{1,\alpha}(A) = \lceil_{\alpha,\beta}(A)$

2) $\lfloor^{\frac{\beta}{1-\alpha}}_{\alpha,1}(A) = \lfloor_{\alpha,\beta}(A)$

Proof It is clear from definition.

Definition 2.2. Let X be a set and $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\} \in IFS(X)$, $\alpha,\beta,\omega,\theta \in [0,1]$. We define the following operator:

$$E^{\omega,\theta}_{\alpha,\beta}(A)=\{<x,\beta((1-(1-\alpha)(1-\theta))\mu_A(x)+(1-\alpha)\theta\nu_A(x)+(1-\alpha)(1-\theta)\omega),$$

$$\alpha((1-\beta)\theta\mu_A(x)+(1-(1-\beta)(1-\theta))\nu_A(x)+(1-\beta)(1-\theta)\omega)>: x\in X\}$$

Proposition 2.2. Let X be a set and $A \in IFS(X)$, $\alpha,\beta,\omega,\theta \in [0,1]$. Then the following statement hold:

$$E^{\omega,\theta}_{\alpha,\beta}(A^c) = E^{\omega,\theta}_{\beta,\alpha}(A)^c$$

Proof It is clear from definition.

Proposition 2.3. Let X be a set and $A \in IFS(X)$, $\alpha,\beta,\omega,\theta \in [0,1]$. If $\beta \le \alpha$ then

$$E^{\omega,\theta}_{\alpha,\beta}(A) \subseteq E^{\omega,\theta}_{\beta,\alpha}(A)$$

Proof If we use $\beta \le \alpha$ then

$$\beta \le \alpha \Rightarrow \beta(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta)) \le \alpha(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta))$$
$$\Rightarrow \beta(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta))+\alpha\beta(\mu_A(x)+\theta\mu_A(x)-\theta\nu_A(x))$$
$$\le \alpha(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta))+\alpha\beta(\mu_A(x)+\theta\mu_A(x)-\theta\nu_A(x))$$

Then we can say $E^{\omega,\theta}_{\alpha,\beta}(A) \subseteq E^{\omega,\theta}_{\beta,\alpha}(A)$.

Proposition 2.4. Let X be a set and $A \in IFS(X)$, $\alpha,\beta,\omega,\theta \in [0,1]$. If $\omega \le \theta$ then

$$E^{\omega,\theta}_{\alpha,\beta}(A) \subseteq E^{\theta,\omega}_{\alpha,\beta}(A)$$

Proof: It is clear from definition.

Definition 2.3. Let X be a set, $A \in IFS(X)$ and $\alpha,\beta \in [0,1]$. We define the following operator:

$$B_{\alpha,\beta}(A) = \left\{ \begin{array}{l} \langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha((1-\beta)\mu_A(x) + \nu_A(x)) \rangle \\ : x \in X \end{array} \right\}$$

Definition 2.4. Let X be a set, $A \in IFS(X)$ and $\alpha,\beta,\omega \in [0,1]$. We define the following operator:

$$\wr_{\alpha,\beta}(A) = \left\{ \langle x, \beta(\mu_A(x) + (1-\beta)\nu_A(x)), \alpha((1-\alpha)\mu_A(x) + \nu_A(x)) \rangle : x \in X \right\}$$

Theorem 2.2. Let X be a set, $A \in IFS(X)$ and $\alpha,\beta \in [0,1]$. Then the following statements hold:

$$B_{\alpha,\alpha}(A) = \wr_{\alpha,\alpha}(A)$$

Proof: It is clear from definition.

Theorem 2.3. Let X be a set, $A \in IFS(X)$ and $\alpha,\beta,\omega \in [0,1]$. The following statements are satisfied:

1) $\lceil^{\omega}_{\alpha,\beta}(A^c) = \lfloor^{\omega}_{\beta,\alpha}(A)^c$

2) $\lfloor^{\omega}_{\alpha,\beta}(A^c) = \lceil^{\omega}_{\beta,\alpha}(A)^c$

3) $\wr_{\alpha,\beta}(A^c) = \wr_{\beta,\alpha}(A)^c$

Proof (1) From definitions of these operators and complement of an intuitionistic fuzzy set we get that,

$$\lfloor^{\omega}_{\beta,\alpha}(A)^c = \{< x, \beta((1-\alpha)\mu_A(x) + \nu_A(x)), \alpha(\beta\mu_A(x) + \omega - \omega\beta) >: x \in X\}$$

and

$$\lceil^{\omega}_{\alpha,\beta}(A^c) = \left\{ \langle x, \beta(\nu_A(x) + (1-\alpha)\mu_A(x)), \alpha(\beta\mu_A(x) + \omega - \omega\beta) \rangle : x \in X \right\}$$

So, we can say $\lceil^{\omega}_{\alpha,\beta}(A^c) = \lfloor^{\omega}_{\beta,\alpha}(A)^c$.

(2) It is clear from definition.

(3) If we use definitions then we get

$$\wr_{\alpha,\beta}(A^c) = \left\{ \langle x, \beta(\nu_A(x) + (1-\beta)\mu_A(x)), \alpha((1-\alpha)\nu_A(x) + \mu_A(x)) \rangle : x \in X \right\}$$

and

$$\wr_{\beta,\alpha}(A)^c = \left\{ \langle x, \beta((1-\beta)\mu_A(x) + \nu_A(x)), \alpha(\mu_A(x) + (1-\alpha)\nu_A(x)) \rangle : x \in X \right\}$$

So, we can say; $\wr_{\alpha,\beta}(A^c) = \wr_{\beta,\alpha}(A)^c$

Theorem 2.5. Let X be a set and $A \in IFS(X)$, $\alpha,\beta \in [0,1]$. If $\alpha \ge \frac{1}{2}$, $\beta \le \frac{1}{2}$ then

$$B_{\alpha\beta}(B_{\beta\alpha}(A)) \subseteq B_{\beta\alpha}(B_{\alpha\beta}(A))$$

Proof If we use $\alpha \ge \frac{1}{2}$ and $\beta \le \frac{1}{2}$ then we get,

$$(1-2\alpha) \le (1-2\beta) \Rightarrow \beta^2(1-2\alpha)(\mu_A(x) + \nu_A(x)) \le \alpha^2(1-2\beta)(\mu_A(x) + \nu_A(x))$$

So,

$$\begin{array}{l} \alpha\beta\mu_A(x) + \alpha\beta(1-\beta)\nu_A(x) + \beta^2(1-\alpha)^2\mu_A(x) + \beta^2(1-\alpha)\nu_A(x) \\ \le \alpha\beta\mu_A(x) + \alpha\beta(1-\alpha)\nu_A(x) + \alpha^2(1-\beta)^2\mu_A(x) + \alpha^2(1-\beta)\nu_A(x) \end{array}$$

and

$$\begin{array}{l} \alpha^2(1-\beta)\mu_A(x) + \alpha^2(1-\beta)^2\nu_A(x) + \alpha\beta(1-\alpha)\mu_A(x) + \alpha\beta\nu_A(x) \\ \ge \beta^2(1-\alpha)\mu_A(x) + \beta^2(1-\alpha)^2\nu_A(x) + \alpha\beta(1-\beta)\mu_A(x) + \alpha\beta\nu_A(x) \end{array}$$

with these inequalities $B_{\alpha\beta}(B_{\beta\alpha}(A)) \subseteq B_{\beta\alpha}(B_{\alpha\beta}(A))$.

As a consequence of above theorem, we can get easily the following propositions;

Proposition 2.5. Let X be a set and $A \in IFS(X)$, $\alpha,\beta \in [0,1]$. Then,

$$B_{\alpha\beta}(A^c) = B_{\beta\alpha}(A)^c$$

Proposition 2.6. Let X be a set and $A \in IFS(X)$, $\alpha,\beta,\omega \in [0,1]$. Then,

1) $E^{\omega,0}_{\alpha,\beta}(A) = Z^{\omega}_{\alpha,\beta}(A)$

2) $E^{\omega,1}_{\alpha,\beta}(A) = B_{\alpha,\beta}(A)$

3) $E^{0,0}_{\alpha,\beta}(A) = G_{\alpha\beta,\alpha\beta}(A)$

4) $E^{1,0}_{\alpha,\beta}(A) = E_{\alpha,\beta}(A)$

5) $E^{0,0}_{1,0}(A) = \varnothing$

6) $E^{0,0}_{0,1}(A) = X$

Proposition 2.7. Let X be a set and $A \in IFS(X)$, $\alpha,\beta,\omega \in [0,1]$. Then,

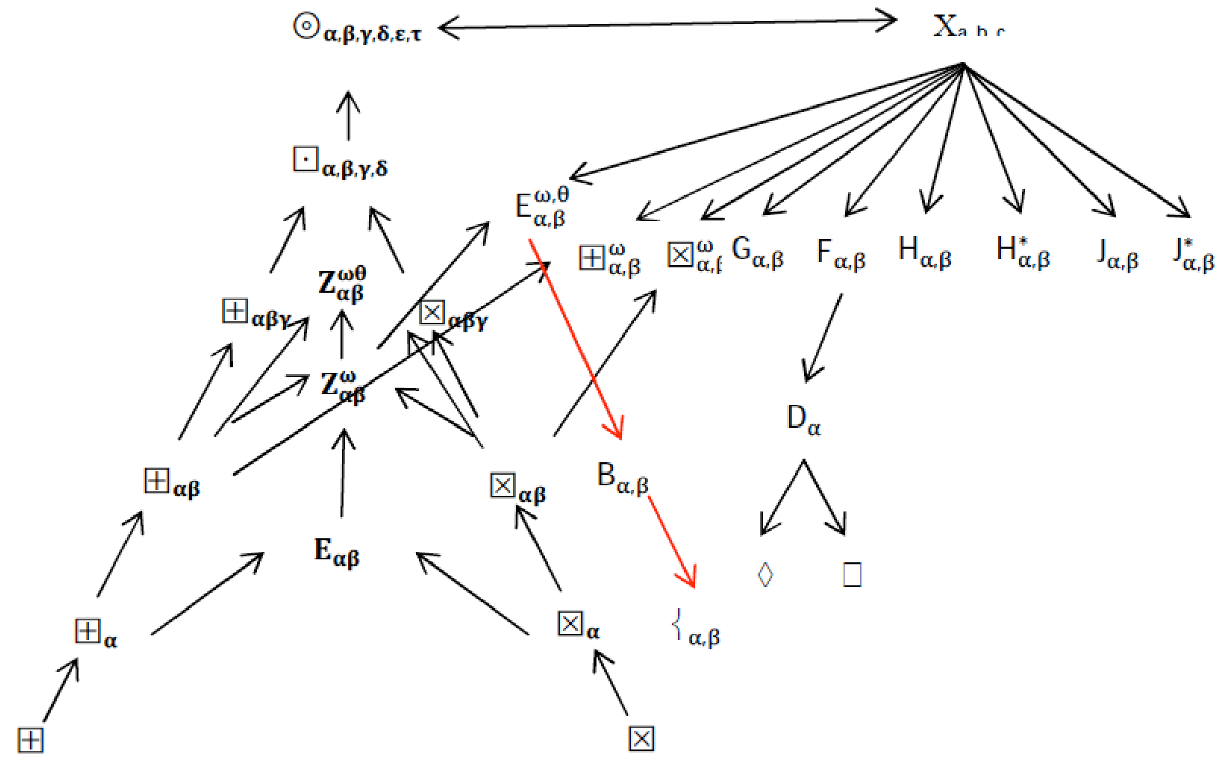

Figure 3:

1) $E^{1,0}_{\alpha,1}(A) = \lfloor_{\alpha}(A)$

2) $E^{\omega,0}_{\alpha,1}(A) = \lfloor_{\alpha,\omega,(1-\alpha)}(A)$

3) $E^{\omega,0}_{1,\beta}(A) = \lceil_{\beta,\omega(1-\beta)}(A)$

4) $E^{\omega,\theta}_{1,1}(A) = A$

5) $E^{1,0}_{1,\beta}(A) = \lceil_{\beta}(A)$

6) $E^{\omega,1}_{\alpha,1}(A) = B_{\alpha,1}(A)$

7) $E^{1,1}_{1,\beta}(A) = B_{1,\beta}(A)$

From the above properties, it is easily show that the operators $\lceil^{\omega}_{\alpha,\beta}$, $\lfloor^{\omega}_{\alpha,\beta}$, $E^{\omega,\theta}_{\alpha,\beta}$, $B_{\alpha\beta}$ and $\{_{\alpha\beta}$ which are defined in this paper are both one type and two type operators. From the above discusssion and the common properties of these operators with the one and two type operators, we give the following definition for the classification;

Definition 2.5. Let X be a set and Υ be a modal operator of Intuitionistic Fuzzy Set on X. If Υ is both one type and two type modal operator then it is called uni-type modal operator of Intuitionistic Fuzzy Set on X.

From that fundemental properties we get the last diagram of all (one/two/uni-type) modal operators on Intuitionistic Fuzzy Sets as in Figure **3**;

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