On Voltage and Power Indicators for Thermal Noise in Metals

Eugen Grycko* and Werner Kirsch

Department of Mathematics and Computer Science, University of Hagen, Universitätsstrasse 1, D-58084 Hagen, Germany

Abstract: Recently a formula for the variance of the thermal voltage between the ends of a conductor has been proposed. We discuss this formula and present numerical values of theoretic voltage and power indicators for a selection of metals. The values obtained for copper are compared with some empirical data.

AMS Subject Classification: 83B30, 83B31.

Keywords: Drude model, electron gas, amplitude of noise voltage.

1. INTRODUCTION

In [1-5] we are concerned with the thermal voltage signal in metals. Methodically, we base on mathematical models that can be evaluated by the application of statistical inference to simulated data. We try to establish bridges between mathematical model, computer experiment and empirical measurements. Sometimes it turns out that this kind of bridges can be based on a comparatively modest experimental instrumentation.

According to a Drude model of the valence electron gas (cf. [1]) and according to our experimental experience a long thin wire is required for the generation of a noise voltage signal which can be identified on the screen of an oscilloscope. Based on this model we conjecture that the amplitude of the noise signal depends in particular on the material of the wire. Unfortunately, we had such problems with the acquisition of a thin wire made out of Mn, Ti, Sc, Ga, Lu, Zr or Y that we cannot offer any empirical evidence for our conjecture. There are nevertheless reasons for the hope that a popularization of appropriate long thin isolated wires on the market would enable the involved researchers to develop applications of the phenomenon of noise voltage for Energy Harvesting (cf. [6,7]).

2. TWO THEORETIC INDICATORS OF THE NOISE LEVEL

In [2] a Drude model of the valence electron gas in metals is considered. N valence electrons within a piece of a metal are interpreted as non-interacting mass points of mass $m = 9.10938291 \cdot 10^{-31}\,kg$ whose velocities $v^{(1)}(t),\ldots,v^{(N)}(t)$ at a time point $t \geq 0$ are modeled as stochastically independent random vectors that are distributed according to the 3-dimensional centered Gaussian distribution

$$N(0,\sigma^2) \otimes N(0,\sigma^2) \otimes N(0,\sigma^2)$$

whose variance $\sigma^2 > 0$ can be interpreted physically as

$$\sigma^2 = \frac{k_B \cdot T}{m} \tag{2.1}$$

where k_B and T denotes the Boltzmann constant and the temperature, respectively.

In [1] a formula for the variance $\mathbb{V}(U(t))$ of the thermal noise voltage signal is derived which can be modeled as a trajectory of a stationary stochastic process $(U(t))_{t \geq 0}$, cf. formula (5.5) in [1]. This formula involves an integral whose approximation yields:

$$\mathbb{V}(U(t)) \approx \varrho e^2 s^2 \sigma^2 \cdot \frac{L}{a^2}. \tag{2.2}$$

The symbols appearing in (2.2) are explained in Table **1**.

Table 1: Legend for (2.2)

ϱ	Material specific density of the electron gas
e	Modulus of the charge of an electron in C
s	Resistivity of the metal in $\Omega \cdot m$
σ^2	Temperature dependent variance of the velocity distribution
L	Length of the wire in m
a^2	Cross-section area of the wire in m^2

Let us now discuss the quantities in (2.2).

*Address correspondence to this author at the Department of Mathematics and Computer Science, University of Hagen, Universitätsstrasse 1, D-58084 Hagen, Germany; E-mail: Eugen.Grycko@fernuni-hagen.de

The modulus $e = 1.6021765 \cdot 10^{-19} C$ of the electric charge of an electron is an universal constant. The values s of the resistivity of metals are generally available. For the determination of ϱ we apply formula (2.3) which requires the knowledge of the following material specific data that we adopt from Wikipedia:

1. Mass density $\bar{\varrho}$ of the metal in kg/m^3

2. Number v of valence electrons per atom

3. The relative atomic mass m_r.

A standard reasoning yields:

$$\varrho = \frac{\bar{\varrho} \cdot N_A \cdot v}{m_r} \tag{2.3}$$

where $N_A = 6.022 \cdot 10^{26} kg^{-1}$ denotes the Avogadro number.

As an indicator of the strength of the voltage signal between the ends of a metallic wire we propose the dispersion.

$$\bar{U} := \left(\mathbb{V}(U(t))\right)^{\frac{1}{2}} \approx e \cdot s \cdot \sigma \cdot \left(\frac{\varrho \cdot L}{a^2}\right)^{\frac{1}{2}}. \tag{2.4}$$

$\bar{U}$ can also be interpreted as the energetic value of the alternating voltage $(U(t))_{t \geq 0}$.

From (2.4) it follows that $\bar{U}$ depends for a given fixed temperature $T = 300K$ on the material and on the geometric form of the conductor. We standardize the form of the metallic conductor according to,

$$L = 1.5 \times 10^4 \text{ m} \quad \text{and} \quad a^2 = \pi \cdot r^2 \tag{2.5}$$

where $r = 6.25 \cdot 10^{-5} w$ m, which means that we consider a wire of length $L = 15000\, m$ and diameter 0.125 mm.

It is well-known that the electric resistance of the considered wire is given by:

$$R = s \cdot \frac{L}{a^2}. \tag{2.6}$$

As a natural power indicator $\bar{P}$ for the thermal noise signal we define,

$$\bar{P} := \frac{\mathbb{V}(U(t))}{R} \approx \varrho \cdot e^2 \cdot s \cdot \sigma^2 \tag{2.7}$$

(cf. (2.2) and (2.6)).

While the voltage indicator $\bar{U}$ is proportional to the square root $\sqrt{L}$ of the wire length L, the power indicator $\bar{P}$ is independent of the geometric form of the conductor.

In Table **2** we present the numerical values of $\bar{P}$ and of $\bar{U}$ for a selection of metals; the determination of these values is based on (2.2), (2.4), (2.5) and (2.7).

Table 2: Theoretic Power and Voltage Indicators

Metal	Power indicator/ μW	Voltage indicator/mV
Ag	0.109	45.88
Al	0.595	143.2
Au	0.152	64.19
Ba	1.192	693.6
Be	1.038	213.2
Ca	0.183	86.35
Cd	0.786	264.3
Co	1.324	316.9
Cr	1.214	429.5
Cu	0.166	58.16
Fe	1.903	47.16
Ga	4.823	1261.6
Hf	3.467	1181.2
In	1.123	338.9
Ir	0.777	210.9
K	0.115	100.4
Li	0.501	237.9
Lu	4.599	1803.8
Mg	0.441	153.4
Mn	26.55	6817.4
Mo	0.402	161.5
Na	0.141	90.4
Nb	0.985	426.6
Ni	0.739	249.5
Os	1.366	365.7
Pb	3.198	901.8
Pt	0.811	321.7
Rb	0.161	158.4
Re	3.061	847.4
Rh	0.367	139.0
Ru	0.614	230.3
Sc	5.242	1892.5

Table 2(Contd.....)

Metal	Power indicator/ μW	Voltage indicator/mV
Sn	2.005	530.8
Sr	0.559	299.4
Ta	1.697	519.9
Ti	5.554	1684.0
Tl	2.199	695.6
V	3.260	883.6
W	0.777	223.3
Y	4.211	1746.9
Zn	0.905	249.5
Zr	4.227	1470.9

3. THE SEARCH FOR EMPIRICAL PENDANTS OF THE THEORETIC VALUES

A careful look at Table **2** reveals that both indicators depend on the material. The power indicator attains its highest values for Mn, Ti, Sc, Ga, Lu, Zr and Y. Since the acquisition of a long thin wire made out of these metals is difficult, we are contented with an ordinary copper wire whose length and diameter are specified in (2.5). The corresponding amplitude $\hat{U}$ of the noise voltage signal can be measured on the screen of an oscilloscope.

In an experiment resistors of varying resistance R_v are connected with the copper wire and the realized amplitude $\hat{U}$ of the noise signal is measured. The observed values are presented in Table **3**.

For the determination of the values in the last column of Table **3** the empirical power indicator $\hat{P} := \frac{\hat{U}^2}{R_v}$ is applied.

An attentive look at Table **3** reveals that the measured value of the voltage amplitude $\hat{U}$ attains for a resistance $R_v = 10^6\,\Omega$ a value similar to the theoretic voltage indicator for copper (cf. Table **2**). The value of the empirical power indicator $\hat{P}$ increases if the value R_v of the resistance of the connected resistor is reduced. Table **3** suggests, moreover, that the theoretical power indicator $\bar{P}$ overestimates the available noise power by a factor of about 14.

Table 3: Empirical Data for Copper Wire

Resistance R_v / Ω connected	Voltage amplitude/mV	Empirical power indicator/nW
10^6	35	1.2
$5 \cdot 10^5$	30	1.8
$2 \cdot 10^5$	20	2.0
10^5	18	3.2
$5 \cdot 10^4$	12	2.9
$2 \cdot 10^4$	8.0	3.2
10^4	7.0	4.9
$5 \cdot 10^3$	6.0	7.2
$2 \cdot 10^3$	4.0	8.0
10^3	3.0	9.0
$5 \cdot 10^2$	2.2	9.7
$2 \cdot 10^2$	1.2	7.2
10^2	1.1	12.1

We may assume that the thermal noise in the probe interferes with the electro-smog-induced noise yielding a cumulated value $\hat{U}$. We nevertheless point out that a general availability of long thin wires made out of Mn, Ti, Sc, Ga, Lu, Zr and Y, respectively, would enable us to widen our empirical explorations into the discussed aspects of noise voltage.

ACKNOWLEDGEMENTS

The authors would like to thank Hans Konrad Knörr from Hagen for valuable comments concerning the subject of the present contribution. We also appreciate the technical support of Hayk Asatryan.

REFERENCES

[1] E. Grycko, W. Kirsch, M. Könenberg, J. Li, T. Mühlenbruch, J. Rentmeister. Thermal noise in a modified Drude Model. *Int J Pure Appl Math* 2009; 54(4): 551-561.

[2] E. Grycko, W. Kirsch, T. Mühlenbruch. Amplification of thermal noise by an electrostatic field. *Int J Pure Appl Math* 2010; 60(2): 187-192.

[3] E. Grycko, W. Kirsch, T. Mühlenbruch. Some quantum mechanical evidence for the amplification of thermal noise in

an electrostatic field. *Int J Pure Appl Math* 2011; 69(4): 437-507.

[4] E. Grycko, W. Kirsch, T. Mühlenbruch. On the thermal voltage signal in a virtual nano-conductor. *Int J Pure Appl Math* 2013; 87(2): 247-260. https://doi.org/10.12732/ijpam.v87i2.5

[5] E. Grycko, W. Kirsch, T. Mühlenbruch. On the thermal angular momentum of the electron gas. *Seminarberichte Mathematik,* Band 2014; 86: 119-128.

[6] K. Dembowski. Energy Harvesting for the microelectronics. VDE Verlag, Berlin, Offenbach 2011.

[7] O. Kanoun, J. Wallaschek. *Energy Harvesting.* expert verlag, Renningen 2008.

Received on 08-01-2017 Accepted on 25-04-2017 Published on 30-07-2017

DOI: http://dx.doi.org/10.15377/2409-5761.2017.04.01.2