Modified Effectiveness Model for Crossflow Packed-Bed Humidifiers

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Abstract: A model for the energy effectiveness of cross-flow packed-bed humidifiers is developed and compared with experimental data. The effectiveness of the humidifier is defined as the sensible and latent heat transferred to the air to the maximum total energy available to the humidified air. Simultaneous heat and mass transfer models; available in the literature and originally developed forwent counter flow cooling towers, are utilized to develop a modified model in this paper. The main modification added to that model is a definition of the heat capacity ratio that combines the simultaneous heat and mass transfer transport in that cross-flow packed-bed humidifiers. It was found that the mixed-mixed crossflow heat exchanger effectiveness relationship developed by Kays and London agrees well with the experimental data with an average deviation of less than 10%. The mixed-unmixed correlation developed by Kays and London [12] which was recommended by Jaber and Webb is found to have larger deviation from the experimental measurements occurred at higher heat capacity ratios.

Keywords: Humidifier, Cross-flow, Packed-bed, Effectiveness, Analytical model, Heat capacity ratio.

1. INTRODUCTION

Humidifiers are used extensively in many commercial and industrial applications. Their uses are numerous for HVAC systems, food storage, medical equipment, etc. when specific humidity requirements must be met. There are many humidification methods such as, steam injection, packed bed humidifiers, ultrasonic, air washers, and spray humidifiers [1]. Like many technologies, it is critical that the device is designed for the application in question which requires expressions that accurately describe the process. Humidifiers operate using a simple principle in which water is dispersed into an area with unsaturated air, the water then diffuses into the air resulting in an increase in humidity [2]. Since the method of operation is like that of cooling towers, with slight difference in the process goal, developing a model for humidifiers based on wet cooling towers model, is a good starting point.

At the present time, the analytical models for humidifiers are based on Merkel's analysis which is used for wet cooling towers. The Merkel model makes three major assumptions to simplify the system of equations. The first assumption is that the dimensionless Lewis number of the process is unity. The Lewis number is a dimensionless parameter used to describe the ratio of the system thermal diffusivity to mass diffusivity. When Lewis number is equal to one, the system is balanced meaning the thermal and mass diffusion rates are equal [3]. The second assumption is that the evaporation of water is neglected resulting in non-conserved mass of water in the system [4]. This indicates that the change in the air specific humidity is zero based on this assumption. The effect the evaporation of water has on the mass flow rate could be neglected within the energy balance. Braun *et al*. [5] estimated that the water loss due to evaporation in cooling towers ranges from 1% to 4% of the feedwater inlet. However, this simplification could result in greater inaccuracies at higher air temperatures [5, 6]. The third assumption is the linear relationship between the enthalpy of saturated air and the temperature which is not accurate at large temperature range [4] (range is the temperature difference between the water inlet and outlet).

Poppe and Rogener [7] developed a model for the wet cooling tower which does not use all of the assumptions that Merkel made in his analysis. Their model allows for the air to be unsaturated, saturated, or supersaturated at the exit. However, this model could be solved only numerically and does not lead to an analytical solution. Jaber and Webb [8] further modified the Merkel model where they defined the energy effectiveness of a cooling tower similar to that of a heat exchanger. Their energy effectiveness model [8] has a good agreement in practice with experimental data of wet cooling towers. However, Sharqawy *et al*. [2] obtained new experimental data of cross flow packed-bed humidifier and the calculated performance was compared with Jaber and Webb [8] predicting model. It was found that their model, has large deviation with the

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experimental data at lower heat capacity ratios [2] (heat capacity of water to air) but in good agreement at higher capacity ratio. When applied to crossflow packed-bed humidifiers their effectiveness model has up to 30% deviation at low heat capacity ratio (at about 0.5) and this deviation declines to about 6% at higher capacity ratio (at about 1.0).

There are some simplified analytical models in the literature for wet cooling towers that could be also used for humidifiers; considering the aim of humidifying cold air with hot water and not cooling the hot water with cooled air (as in the cooling tower). These analytical models assume that the enthalpy of the saturated air is linearly related to the water temperature to allow solving the model equations analytically. Kloppers and Kröger [9, 10] investigated the relationship between the saturated air enthalpy and water temperature for counter-flow wet cooling towers. Their analysis indicates that there is a noticeable discrepancy in the enthalpy change, especially at the hot water side. They concluded that Poppe's analysis method is better suited at giving a more accurate picture than that of the or Merkel model. Mansour and Fath [11] defined a new variable for air humidifier which is the ratio between a fictitious specific heat of air and the water specific heat. The values of the fictitious specific heat of air has linear relationship with the saturated air enthalpy at the water temperature. Their approach aimed to find an effective sensible heat ratio that account for both the heat and mass transfer processes, which is then used to determine the effectiveness of the humidifier. This method reduces the deviation in Merkel's approach but is still not as accurate as a higher order non-linear correlation [11].

The objective of this paper is to develop an analytical model for cross-flow packed bed humidifier to assess the inaccuracies associated with Merkel's model assumptions. Efforts are focused in providing experimental results and subsequent analytical models which agree within reasonable tolerances with collected data. As per the throughout review, many of the existing models used to design humidification systems have large margins of error. The inherent difficulty of deriving an accurate method for the heat and mass transfer in the humidifier will be carefully thought out. Using the already derived relationships for crossflow heat exchangers, the same correlation will be applied for humidifiers after modifying the correlation parameters. The modified model will be validated using the experimental data provided by Sharqawy *et al*. [2].

2. MATHEMATICAL MODEL

Figure **1** shows a control volume of a crossflow humidifier where water enters at the top of a fill material and moving down in the y-direction by gravity, while air is flowing across the fill material in the z-direction. The following are the assumptions made to simplify the model.

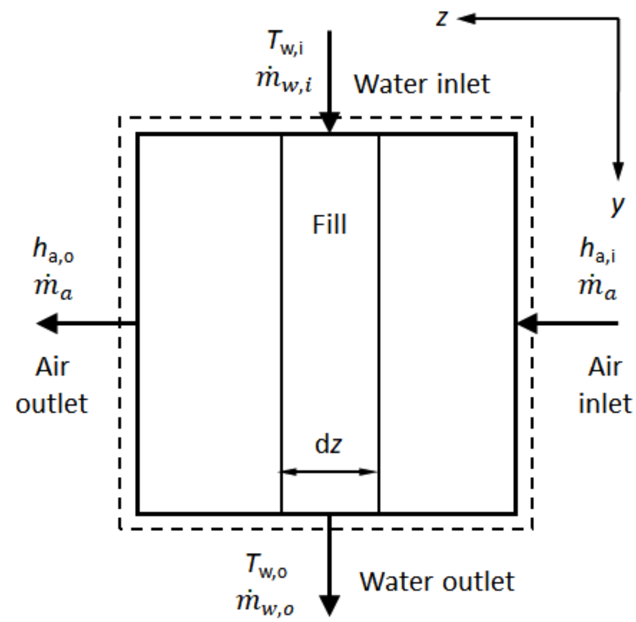

Figure 1: Control volume of a crossflow humidifier.

- Negligible heat transfer through walls
- Negligible water loss in water through humidification/evaporation
- Negligible heat input from pump or fan to the water or air streams

Equation (1) is an overall energy balance on the control volume which considers both sensible and latent heat transfer in the water and air streams. The moist air enthalpy in Eq. (1) considers the specific enthalpy of the dry air and water vapor carried by the air stream.

$$\dot{m}_w c_{p,w} dT_w = -\dot{m}_a dh_a \quad (1)$$

Equation 1 can be rewritten after introducing the saturation specific heat, C_s, as defined in Eq. (3).

$$\dot{m}_w c_{p,w}\, dh_{sat,w} = -\dot{m}_a C_s dh_a \quad (2)$$

where C_s is the specific heat capacity of the saturated air at the water temperature and is approximately calculated using the average slope between the inlet and outlet saturated air enthalpy at the water temperature.

$$C_s = \frac{dh_{sat,w}}{dT_w} = \frac{h_{sat,w,in} - h_{sat,w,out}}{T_{w,in} - T_{w,out}} \quad (3)$$

Applying an energy balance of the air stream and using an average mass transfer coefficient (h_D) yields,

$$\dot{m}_a dh_a = -h_D A\left(h_{sat,w} - h_a\right) dz \quad (4)$$

The solution of equations (2) and (4) gives the exit water temperature and exit air enthalpy which can be used to determine the overall heat transfer after an overall energy balance on any stream. The overall heat transfer can also be written in terms of the air effectiveness as given in Eq. (5).

$$\dot{Q} = \varepsilon_a \dot{m}_a (h_{w,in} - h_{a,in}) \quad (5)$$

where the air effectiveness of the humidifier is defined as follows:

$$\varepsilon_a = \frac{(h_{a,out} - h_{a,in})}{(h_{sat,w,in} - h_{a,in})} \quad (6)$$

It is important to emphasis here that in cooling towers, water-based effectiveness is commonly used for the performance since the purpose is to cool water. However, in humidifiers, air-based effectiveness is commonly used since the target is to humidify the air. To analytically determine the effectiveness, Jaber and Webb [8] recommended using Kays and London [12] equation of the unmixed/unmixed crossflow heat exchanger which is given by Eq. (7) to determine the effectiveness.

$$\varepsilon_{U-U} = 1 - \exp\left[\frac{NTU^{0.22}}{C_r}\left[\exp(-C_r NTU^{0.78}) - 1\right]\right] \quad (7)$$

where C_r is the heat capacity ratio which was given by Braun *et al*. [5] as follows:

$$C_r = \frac{\dot{m}_a (h_{w,in} - h_{w,out})}{\dot{m}_{w,in} c_{p,w} (T_{w,in} - T_{w,out})} \quad (8)$$

In addition, the number of transfer units, NTU is given by,

$$NTU = \frac{h_D A_s}{\dot{m}_a} \quad (9)$$

Sharqawy *et al*. [2] found that using this definition of the Cr and the unmixed/unmixed correlation for the crossflow heat exchanger (Eq. 7) to determine the effectiveness of the humidifier results in a large deviation with the experimental data measured for cross-flow packed bed humidifier at lower heat capacity ratios. In addition, the definition of the *NTU* as given by Eq. (9), does not allow to determine its value unless the effectiveness is known (*i.e*. a reverse way of using the effectiveness correlation) because the overall mass transfer coefficient is usually not known. Therefore, a method is proposed in the present work to determine the effective number of transfer units using an incremental set of equations to decrease the error in the linearized saturated air enthalpy correlation. This is done by dividing the humidification process curve into *N* number of segments and summing the enthalpy ratio as given by Eq. (10). Equation (10) could be explained as a numerical integrated version of the differential energy balance given by Eq. (4).

$$NTU = \frac{h_D A_s}{\dot{m}_a} = \sum_{i=1}^{N} \frac{\Delta h_{a,in}}{h_{sat,a,in} - h_{a,in}} \quad (10)$$

where *N* is the number of linear piecewise equations. In addition, a modified definition of the heat capacity ratio is proposed and given by Eq. (11). This definition integrates the air enthalpy slope with the water temperature along the water temperature range.

$$C_{r,mod} = \frac{\int_{T_{w\,out}}^{T_{w\,in}} \left(1 + \sqrt{\left(\frac{dh}{dT_w}\right)}\right) dT_w}{(T_{w\,in} - T_{w\,out})} \quad (11)$$

The new definitions of *NTU* and C_r given by equations 10 and 11 require a relationship between the saturated air enthalpy and the temperature. Due to the deviation in model prediction using equation (8) which is taking an average slope for the saturated air enthalpy at larger range, the new correlation is obtained by a best fit polynomial curve of the psychometric data for saturated air at atmospheric pressure of 101.325 kPa. A fourth order polynomial correlation with a correlation coefficient of 99.9% is given by Eq. (12). Figure **2** shows the applied range of this correlation.

$$h_{sat} = 53.971 - 5.72651\,T_w + 0.441192\,T_w^2 - 0.00930861\,T_w^3 + 0.0000904223\,T_w^4 \quad (12)$$

Moreover, the following equation for the mixed-mixed cross flow heat exchanger is used in the present work model as the effectiveness-NTU correlation which is also given in Kays and London [12].

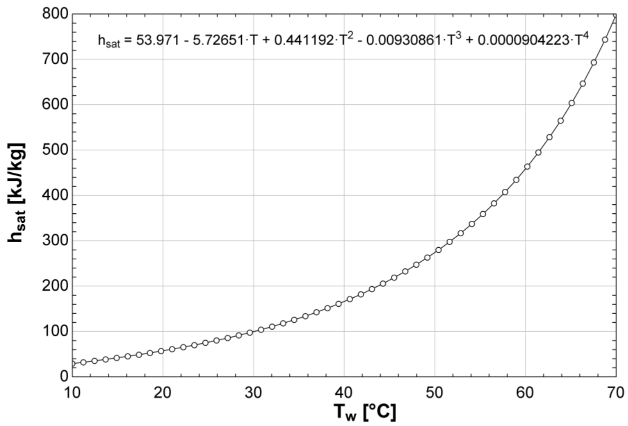

Figure 2: Saturated air enthalpy vs. temperature with a 4th order polynomial correlation.

$$\varepsilon_{M-M} = \frac{NTU}{\left(\frac{NTU}{1-\exp\left(-NTU\right)}\right)} + \left(\frac{C_r NTU}{1-\exp\left(-C_r NTU\right)} - 1\right) \tag{13}$$

Using Eq. 13 together with the new defined *NTU* and C_r in equations (10) and (11) respectively, the effectiveness of the crossflow humidifier can be determined. The following section will present the predicted effectiveness from the proposed model and compare it with experimental data presented by Sharqawy *et al*. [2] for cross flow packed-bed humidifier.

3. RESULTS AND DISCUSSION

The modified heat capacity ratio given by Eq. (11) and the *NTU* definition given by Eq. (10), together with the mixed/mixed effectiveness correlation given by Eq. (13) are used to calculate the effectiveness of the cross-flow humidifier. Figure **3** shows the experimental and predicted effectiveness across a range of NTU and heat capacity ratios. The effectiveness is calculated in Figure **3** using the two correlations of the mixed/mixed and unmixed/unmixed cross flow heat exchanger but with the modified *NTU* and C_r. Figure **4** and Figure **5** show the experimental and predicted effectiveness for different number of fills. The increase in the number of fills indicates a larger thickness of the packing material or fill. The two figures show that both mixed-mixed and unmixed-unmixed effectiveness correlations have good agreement with the experimental effectiveness after using the new definition of the NTU and heat capacity ratio.

In order to determine which of the two effectiveness correlations have better agreement with the measured data at higher temperature range (water temperature difference ΔT_w), the percentage deviation of the effectiveness values is calculated and presented in

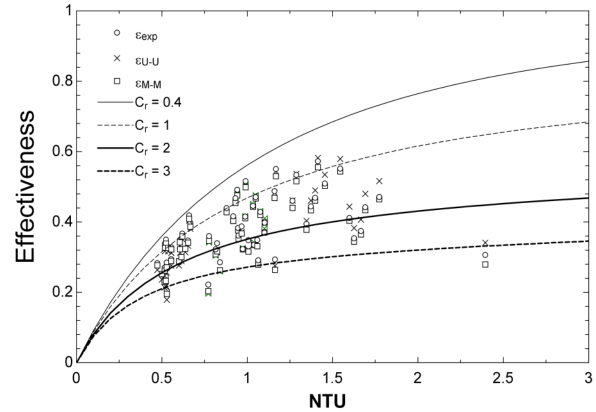

Figure 3: Measured effectiveness data and predicted effectiveness models as a function of NTU and at different heat capacity ratios.

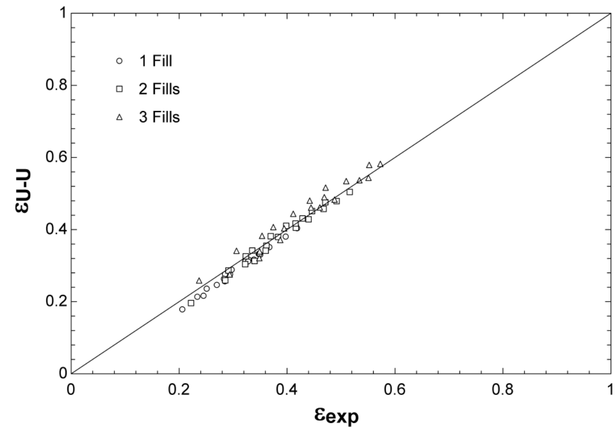

Figure 4: Measured effectiveness vs. crossflow unmixed-unmixed effectiveness model comparison with the modified specific heat capacity.

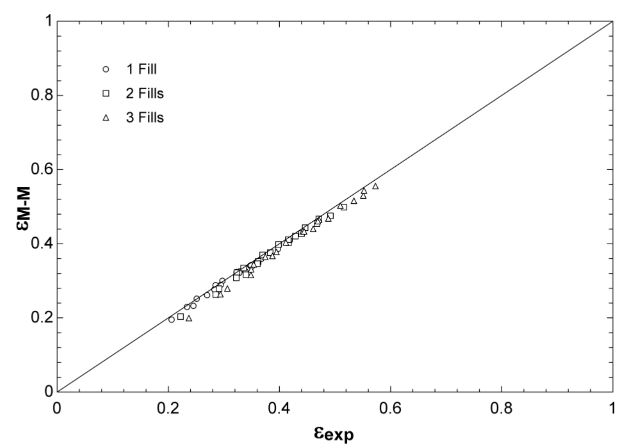

Figure 5: Measured effectiveness vs. crossflow mixed-mixed effectiveness model comparison with the modified specific heat capacity.

Figures **6** – **8**. Figures **6** – **8** illustrate the deviation in effectiveness between the calculated and experimental values for the unmixed and mixed correlations for 1 fill, 2 fills, and 3 fills respectively. Comparing these figures, it is noticed that the mixed-mixed correlation (Eq. 11) has lower deviation and hence better agreement with the experimental effectiveness values compared with the unmixed-unmixed correlation (Eq. 10). The average deviation for the unmixed/unmixed (U-U) crossflow equation is 5.93%, 3.25%, and 5.19% for 1fill, 2 fills, and 3 fills respectively. The mixed/mixed (M-M) crossflow equation has more consistent effectiveness relationship over the three experiments with an average error of 2.26%, 2.93%, and 4.55% for 1 fill, 2 fills, and 3 fills respectively. It was found that the definition of the heat capacity ratio defined by Braun *et al*. [5] (Eq 8) is better at predicting the effectiveness of the humidifier than the one used in Jaber and Webb [8] (Eq. 3) but worse than the proposed one in the presented work (Eq. 11).

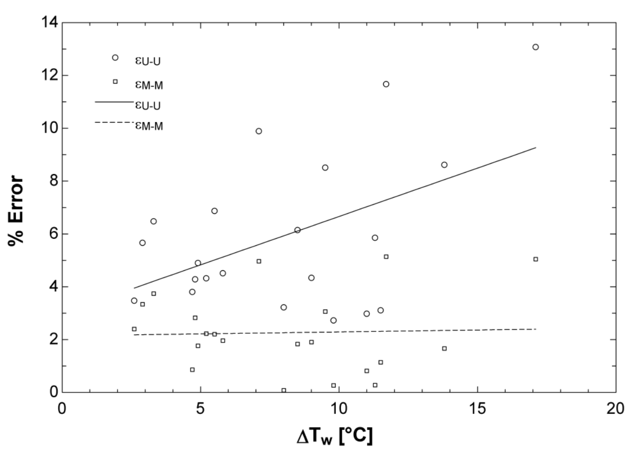

Figure 6: Percent error of the 1 fill experiment as the ΔTw increases.

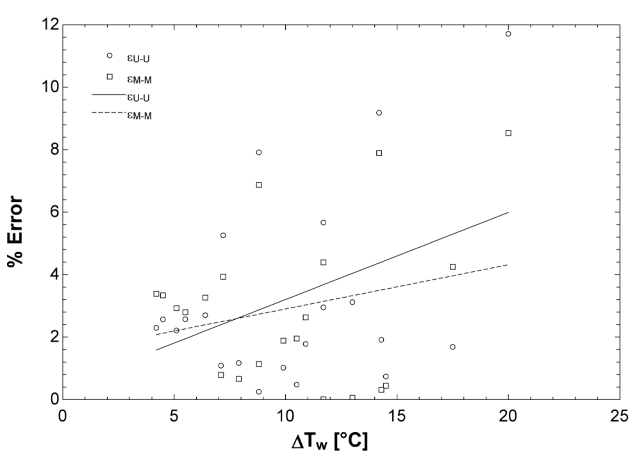

Figure 7: Percent error of the 2 fill experiment as the ΔTw increases.

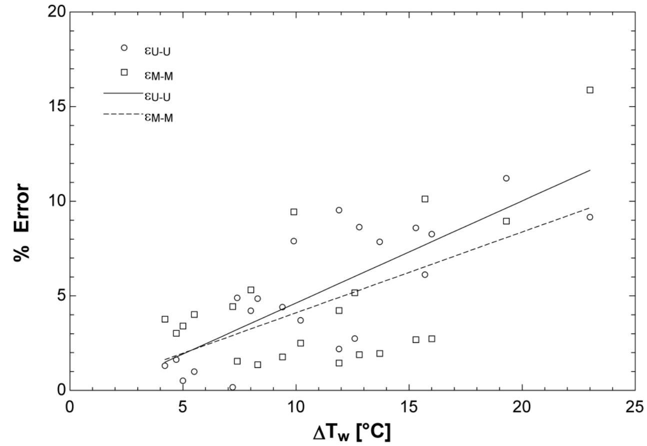

Figure 8: Percent error of the 3 fill experiment as the ΔTw increases.

Figure **9** and Figure **10** compare the predicted effectiveness using the mixed-mixed and unmixed-unmixed correlations with Braun *et al*. [5] definition of the heat capacity ratio (Eq. 8). Although this definition works well in the 1 fill experimental (cross-flow thickness of 10 cm) it starts to deviate in the 2 fills (20 cm thickness) and reaches maximum deviation in the 3 fills (30 cm thickness). This is expected as the average change in water temperature is the highest in the 3 fills case (ΔT_w = 11 compared to 8 in the 1 fill). This could be explained by the effect of the water loss and the linear approximation of the saturated air enthalpy and water temperature correlation becomes more pronounced as the change in water temperature increases as shown in Figure **6-8** which illustrate the effect of on the effectiveness's error. The unmixed-unmixed crossflow equation is more sensitive to the increase of the water temperature difference.

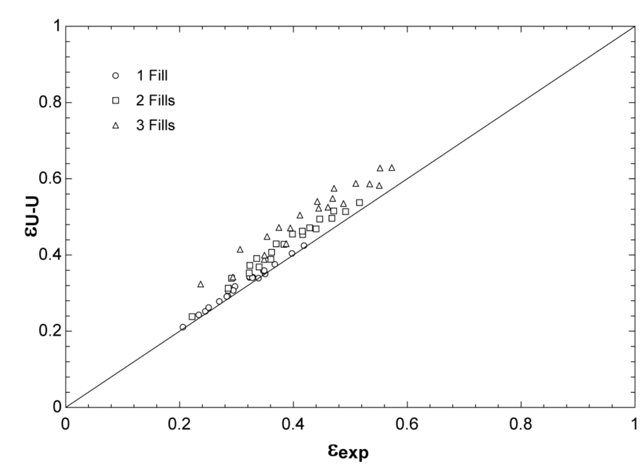

Figure 9: Measured effectiveness vs. crossflow unmixed-unmixed effectiveness model comparison with the Braun *et al.* [5] definition of the specific heat ratio.

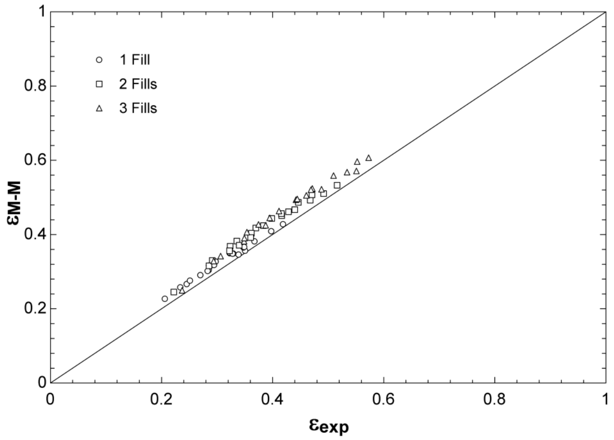

Figure 10: Measured effectiveness vs. crossflow mixed-mixed effectiveness model comparison with the Braun et al. [5] definition of the specific heat ratio.

Figures **11-13** show the percentage error in the calculated effectiveness for both the specific heat ratio correlations for 1, 2, and 3 fills experimental data. These figures show also a fitted linear curve fit obtained of the percentage deviation to see the trend of the error. For the 1 fill experiment the error trend of every model except the U-U unmodified is decreasing with NTU. For the 2 fills experiment showed in Figure **12** the error in the two modified equations decrease linearly with NTU while the opposite is true for the unmodified versions. Finally, in the 3 fills experiment all but the unmodified M-M equation is subject to increases in error as the NTU increases.

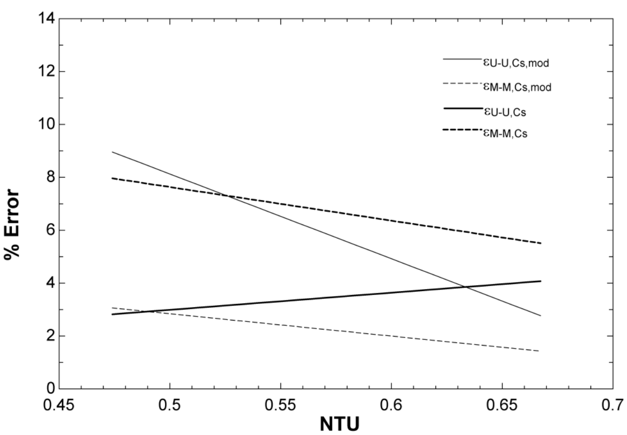

Figure 11: Percent error of the calculated effectiveness for both definitions of specific heat ratio for the 1 fill experiment.

CONCLUSIONS

Using a modified version of Braun *et al*. [5] specific heat ratio, the effectiveness of the cross-flow packed-bed humidifier was tested with two heat exchanger effectiveness relationships. It was found that the cross-

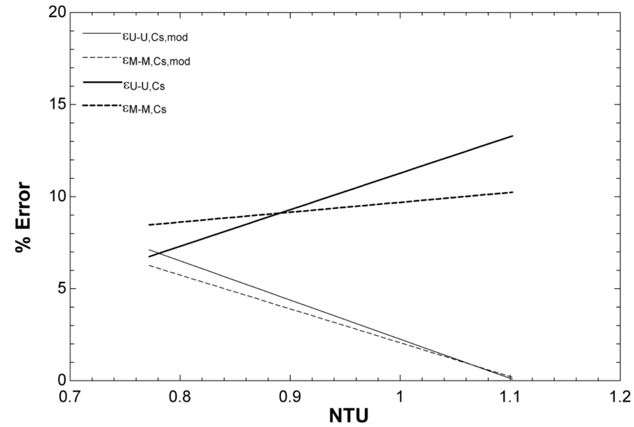

Figure 12: Percent error of the calculated effectiveness for both definitions of specific heat ratio for the 2 fill experiment.

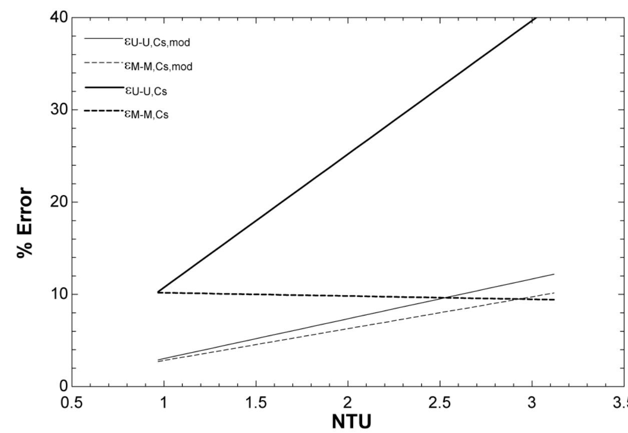

Figure 13: Percent error of the calculated effectiveness for both definitions of specific heat ratio for the 3 fill experiment.

flow equation for mixed-mixed gives better results of lower deviation from the experimental data compared with the unmixed-unmixed one. Crossflow unmixed-unmixed equation has lower error in the short fill or packing data set but has high error in the large fill data set. Average error in the 1 fill, 2 fill, and 3 fills is 5.93%, 3.25%, and 5.19% respectively. The higher error in the three fills could be due to the neglection of the water evaporation in the model and the linear approximation of the enthalpy-temperature correlation. The crossflow mixed-mixed equation had more consistent results across the three different fill experiments. The average error the 1 fill, 2 fill, and 3 fills is 2.26%, 2.93%, and 4.55% respectively.

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NOMENCLATURE

A	cross sectional area	m^2
A_s	surface area	m^2
c_p	specific heat	$J\ kg^{-1}K^{-1}$
C_r	heat capacity ratio	_
C_s	saturated air specific heat	$J\ kg^{-1}K^{-1}$
h	specific enthalpy	$J\ kg^{-1}$
h_D	mass transfer coefficient	$kg\ s^{-1}\ m^{-2}$
$\dot{m}$	mass flow rate	$kg\ s^{-1}$
N	number of linear piecewise equations in Eq. (10)	
NTU	number of transfer units	_
$\dot{Q}$	rate of heat transfer	$J\ s^{-1}$
T	temperature	^oC
y	y-coordinate	
z	z-coordinate	

Greek Symbols

ε	effectiveness	_

Subscript

a	air
avg	average
in	inlet
out	outlet
min	minimum
mod	modified
M-M	Mixed-Mixed
sat	saturation
U-U	unmixed-unmixed
w	water

REFERENCES

[1] ASHRAE Handbook - HVAC Applications, ASHRAE Inc, 2015.

[2] Sharqawy M, Al-Shalawi I, Antar M and Zubair S. Experimental investigation of packed-bed cross-flow humidifier, Applied Thermal Engineering 2017; 117: 584-590. https://doi.org/10.1016/j.applthermaleng.2017.02.061

[3] Kloppers J and Kröger D. The Lewis factor and its influence on the performance prediction of wet-cooling towers, International Journal of Thermal Sciences 2005; 44(9): 879-884, https://doi.org/10.1016/j.ijthermalsci.2005.03.006

[4] Xie Y, Xu Z and Mei N. Evaluation of the effectiveness-NTU method for countercurrent humidifier, Applied Thermal Engineering 2016; 99: 1270-1276. https://doi.org/10.1016/j.applthermaleng.2016.01.112

[5] Braun J, Klein S and Mitchell J. Effectiveness models for cooling towers and cooling coils, ASHRAE Transactions 1989; 95: 164-174.

[6] Sutherland J. Analysis of Mechanical-Draught Counterflow Air/Water Cooling Towers, Journal of Heat Transfer 2009; 105(3): 576-583. https://doi.org/10.1115/1.3245624

[7] Poppe M, Rogener H, Berechnung von Rückkühlwerken, VDI-Warmeatlas 1991; 1-15.

[8] Jaber H and Webb R. Design of cooling towers by the effectiveness-NTU method, Journal of Heat Transfer 1989; 111(4): 837-843. https://doi.org/10.1115/1.3250794

[9] Kloppers J and Kröger D. A critical investigation into the heat and mass transfer analysis of crossflow wet cooling towers, Numerical Heat Transfer 2010; 46: 785-806. https://doi.org/10.1080/104077890504113

[10] Kloppers J and Kröger D. A critical investigation into the heat and mass transfer analysis of counterflow wet-cooling towers, International Journal of Heat and Mass Transfer 2004; 48: 765-777. https://doi.org/10.1016/j.ijheatmasstransfer.2004.09.004

[11] Mansour M and Fath H. A new and practical ε-NTU correlation for the humidification process under different Lewis number, Desalination 2016; 365(3): 72-78. https://doi.org/10.1016/j.desal.2016.05.025

[12] Kays WM, London AL. Compact Heat Exchangers, 3 ed., McGraw-Hill, New York, 1984.

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