Creating Alternatives for Stochastic Water Resources Management Decision-Making Using a Firefly Algorithm-Driven Simulation-Optimization Approach

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Abstract: In solving complex water resources management (WRM) problems, it can prove preferable to create numerous quantifiably good alternatives that provide multiple, disparate perspectives. This is because WRM normally involves multifaceted problems that are riddled with incompatible performance objectives and contain inconsistent design requirements, which are very difficult to quantify and capture when supporting decisions must be constructed. By producing a set of options that are maximally different from each other in terms of their unmodelled variable structures, it is hoped that some of these dissimilar solutions may convey very different perspectives that may serve to address these unmodelled objectives. In environmental planning, this maximally different option production procedure is referred to as modelling-to-generate-alternatives (MGA). In addition, many components of WRM problems possess extensive stochastic uncertainty. This study provides a firefly algorithm-driven simulation-optimization approach for MGA that can be used to efficiently create multiple solution alternatives to problems containing significant stochastic uncertainties that satisfy required system performance criteria and yet are maximally different in their decision spaces. This algorithmic approach is both computationally efficient and simultaneously produces a prescribed number of maximally different solution alternatives in a single computational run of the procedure. The effectiveness of this stochastic MGA approach for creating alternatives in "real world", environmental policy formulation is demonstrated using a WRM case study.

Keywords: Water resources management, modelling-to-generate alternatives, firefly Algorithm.

1. INTRODUCTION

Water resource managers have been confronted by water allocation problems for many decades [1, 2]. Implementing effective water resources management (WRM) has proven to be both notoriously contentious and conflict- laden as the inherent antagonism between multiple municipal, industrial and agricultural water-users has intensified. Increased population shifts and shrinking water supplies have further aggravated the inter-user challenges. These antagonisms provoke additional aggravations when natural conditions become more unpredictable due to changing climatic conditions and as concern for water quantity and quality grows. Poorly-planned water allocation systems can deteriorate into more serious conflicts under detrimental river-flow and climatic conditions. In the past, increasing demand for water was met by the development of new water sources. However, significant economic and environmental costs associated with developing new water sources have rendered this approach unsustainable. The unlimited expansion of water sources is no longer the primary objective in WRM. Instead, for optimum water resource allocation, it is desired to improve the existing water allocation and management in a more equitable, environmentally-benign, and efficient manner by fashioning environmental policy formulation techniques for water allocation under various complexities. Such innovative environmental policy formulation can be extremely problematic, as many components of water systems contain substantial degrees of uncertainty. The prevalence of stochastic uncertainty renders most common decision approaches relatively unsuitable for practical implementation.

Since problems of WRM management generally possess all of the characteristics associated with environmental planning, WRM systems have provided an ideal backdrop for the testing of a wide spectrum of decision support techniques used in environmental decision-making [3-5]. WRM decision-making frequently involves complex problems that possess design requirements which are very difficult to incorporate into any supporting modelling formulations and tends to be plagued by various unquantifiable components [6-13]. Numerous objectives and system requirements readily exist that can never be unambiguously captured during the problem formulation stage [14, 15]. This commonly occurs in "real world" situations where final decisions must be constructed based not only upon clearly articulated specifications, but also upon environmental, political and socio-economic objectives that are either fundamentally subjective or not clearly articulated [16-18].

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Moreover, in public policy formulation, it may never be possible to explicitly convey many of the subjective considerations because there are numerous competing, adversarial stakeholder groups holding diametrically opposed perspectives. Therefore many of the subjective aspects remain unknown, unquantified and unmodelled in the construction of any corresponding decision models. WRM policy formulation can prove even more complicated when the various system components also contain considerable stochastic uncertainties [19, 20]. Consequently, WRM policy determination proves to be an extremely challenging and complicated undertaking [10, 21, 22].

Various ancillary mathematical modelling approaches have been proposed to support environmental policy formulation (see, for example: [4, 7, 11, 14, 23-25]. However, while mathematically optimal solutions may provide the best answers to these modelled formulations, they generally do not supply the best solutions to the underlying real problems as there are invariably unmodelled aspects not apparent during the model construction phase [6, 10, 11, 21, 26-29]. Furthermore, although deterministic optimization-based techniques are designed to create single best solutions, the presence of the unmodelled issues coupled with the system uncertainties and opposition from powerful stakeholders can actually lead to the outright exclusion of any single (even an optimal) solution from further consideration [8, 9, 15, 18-20, 31-33]. Under conflicting circumstances where no universally optimal solution exists, it has been stated that “there are no ideal solutions, only trade-offs” [34] and several underlying behavioural characteristics adopted by decision-makers when faced with such difficulties are outlined in [26].

Within WRM decision-making, there are routinely many stakeholder groups holding completely incongruent standpoints, essentially dictating that policy-makers need to construct decision frameworks that can somehow simultaneously consider numerous irreconcilable points of view [8, 9, 14, 20, 33, 35, 36]. In general, it is considered advantageous to be able to generate a reasonably judicious number of very different alternatives that provide multiple, contrasting perspectives to the specified problem [13, 33, 37-39]. These alternatives should preferably all possess near-optimal objective measures with respect to all of the modelled objective(s) that are known to exist, but be as fundamentally different from each other as possible in terms of the system structures characterized by their decision variables. By generating such a diverse set of solutions, it is hoped that at least some of the dissimilar alternatives can be used to address the requirements of the unknown or unmodelled criteria to varying degrees of stakeholder acceptability. Several approaches collectively referred to as modelling-to-generate-alternatives (MGA) have been developed in response to this multi-solution creation requirement [17, 18, 21, 24, 25, 29, 38-43].

MGA techniques employ a methodical examination of the solution space in order to generate a set of alternatives that are considered good when measured within the modelled objective space while being maximally different from each other in the decision space. The resulting alternatives provide a set of diverse approaches that all perform similarly with respect to the known modelled objectives, yet very differently with respect to any unmodelled issues [13, 43]. Subsequently the policy-makers must conduct comprehensive evaluations of these alternatives to determine which options more closely placate their particular circumstances. Thus, a good MGA process should enable a thorough exploration of the decision space for good solutions while simultaneously allowing for unmodelled objectives to be considered when making final decisions. Consequently, unlike the more customary practice of explicit solution determination inherent in most “hard” optimization methods of mathematical programming, MGA approaches must necessarily be considered as decision support processes.

Deterministic MGA methods are comparatively unsuitable for most WRM policy formulation, since the components of most WRM systems possess considerable stochastic uncertainty [11, 14, 19, 28, 30, 31, 35, 44-46]. Yeomans *et al.* [47] integrated stochastic uncertainty directly into planning using an approach referred to as simulation-optimization (SO). SO is a family of optimization techniques that incorporates inherent stochastic uncertainties expressed as probability distributions directly into its computational procedure [48-50]. To address the deficiencies in deterministic MGA methods, Yeomans [36] demonstrated that SO could be used to generate multiple alternatives which simultaneously incorporated stochastic uncertainties directly into each generated option. Since computational aspects can negatively impact SO’s optimization capabilities, these difficulties clearly also extend into its use as an MGA procedure [7, 20]. Linton *et al*. [4] and Yeomans [20] have shown that SO can be considered an effective, though very computationally intensive, MGA technique for policy formulation. Furthermore, none of these SO-based approaches could ensure that the created alternatives were sufficiently different in decision variable structure from one another to be considered an effective MGA procedure.

In this study, a stochastic MGA procedure is described that efficiently generates sets of maximally different solution alternatives by executing an revised form of the nature-inspired Firefly Algorithm (FA) [5, 51, 52] combined with a co-evolutionary MGA approach [3, 53-57]. Yang [51] has demonstrated that the FA is a more computationally efficient procedure than such commonly used metaheuristics as enhanced particle swarm optimization, genetic algorithms, and simulated annealing. The FA-driven stochastic MGA procedure extends the deterministic approaches in [53-56] by advancing the FA into SO for stochastic optimization and by exploiting the concept of co-evolution within the FA's solution methods to concurrently generate the requisite number of solution alternatives (see [3, 57]. Remarkably, this innovative algorithm can simultaneously generate the overall optimal solution together with *n* maximally different, locally optimal alternatives in a single computational run. Hence, the stochastic FA-driven procedure is computationally efficient for MGA purposes. Using the solution generation framework employed in [57], the effectiveness of this method for WRM purposes is demonstrated using a case study taken from [1] and [2]. More significantly, the practicality of this stochastic MGA FA-driven approach can quite easily be modified to many other stochastic planning systems and, therefore, can be readily adapted to address numerous other applications.

2. MODELLING TO GENERATE ALTERNATIVES

Most mathematical programming algorithms appearing in the optimization literature have concentrated almost exclusively on producing single best solutions for single-objective formulations or, equivalently, generating noninferior solutions for multi-objective problems [10, 13, 17, 43]. While such techniques may efficiently generate solutions to the derived complex mathematical models, whether these outputs actually establish "best" approaches to the underlying real problems has been called into question [6, 10, 17, 21]. In most "real world" decision-making situations, there are numerous system objectives and requirements that are never explicitly included or apparent during the problem formulation [6, 13]. Furthermore, it may never be possible to explicitly express all of the subjective components because there are frequently numerous incompatible, competing, design requirements and, perhaps, adversarial stakeholder groups involved [9, 14, 37]. Therefore, most subjective aspects of a problem remain unquantified and unmodelled in the resultant decision models. This is a common occurrence in situations where final decisions are constructed based not only upon clearly stated and modelled objectives, but also upon more fundamentally subjective socio-political-economic goals and stakeholder preferences [37-39]. Several "real world" examples highlighting these types of incongruent modelling dualities in environmental decision-making are described in [17, 18] and [21].

When unmodelled objectives and unquantified issues exist, unconventional approaches are needed that not only explore the decision space for noninferior sets of solutions, but also examine the decision space for discernibly *inferior* alternatives to the modelled problem. In particular, any search for good alternatives to problems known or suspected to contain unmodelled objectives must focus not only on the non-inferior solution set, but also necessarily on an explicit exploration of the formulation's entire inferior feasible region.

To illustrate the implications of an unmodelled objective on a decision search, assume that the optimal solution for a quantified, single-objective, maximization decision problem is $\boldsymbol{X}^*$ with corresponding objective value $Z1^*$. Now suppose that there exists a second, unmodelled, maximization objective $Z2$ that subjectively reflects some unquantifiable component such as "political acceptability". Let the solution $\boldsymbol{X}^c$, belonging to the noninferior, 2-objective set, represent a potential best compromise solution if both objectives could somehow have been simultaneously evaluated by the decision-maker. While $\boldsymbol{X}^c$ might be viewed as the best compromise solution to the real problem, it would appear inferior to the solution $\boldsymbol{X}^*$ in the quantified mathematical model, since it must be the case that $Z1^c \leq Z1^*$. Consequently, when unmodelled objectives are factored into the decision-making process, mathematically inferior solutions for the modelled problem can prove optimal to the underlying real problem [38-57].

Therefore, when unquantified issues and unmodelled objectives could exist, unorthodox methods are employed to not only search the decision space for noninferior sets of solutions, but also to simultaneously explore the decision space for inferior alternative solutions to the modelled problem. Population-based search techniques such as the FA permit concurrent examinations throughout a decision space and prove to be particularly adept at searching throughout the problem's feasible region.

The principal objective underlying MGA is to produce a manageably small set of alternatives that are quantifiably good with respect to the known modelled objective(s) yet are as different as possible from each other within the decision space. In doing this, the resulting solution set is likely to provide truly different alternatives that all perform somewhat similarly with respect to the modelled objective(s) yet very differently with respect to any unknown unmodelled issues. By generating a set of good-but-different solutions, the decision-makers can explore desirable qualities within the alternatives that may prove to satisfactorily address the various unmodelled objectives to varying degrees of stakeholder acceptability.

To suitably motivate an MGA procedure, it is necessary to apply a more mathematically formal definition to the goals of the MGA process [21, 37, 39]. Suppose the optimal solution to an original mathematical model is $\boldsymbol{X}^*$ with objective value $\boldsymbol{Z}^* = F(\boldsymbol{X}^*)$. The following maximal difference model, subsequently referred to in the paper as problem [P1], can then be solved to generate an alternative solution that is maximally different from $\boldsymbol{X}^*$:

$$Maximize \quad \Delta = \sum_i \left| X_i - X_i^* \right| \qquad \text{(A1)}$$

Subject to: $\boldsymbol{X} \in D$

$| F(\boldsymbol{X}) - \boldsymbol{Z}^* | \leq T$

where Δ represents some difference function (for clarity, shown as an absolute difference in this instance), D is the original mathematical model's feasible domain and T is a targeted tolerance value specified relative to the problem's original optimal objective $\boldsymbol{Z}^*$. T is a user-supplied target that effectively designates how much of the inferior region is to be explored in the search for acceptable alternative solutions.

3. FIREFLY ALGORITHM FOR FUNCTION OPTIMIZATION

While this section provides only an abridged description of the FA procedure, more detailed explanations appear in [3] and [51-57]. The FA is a population-based, nature-inspired metaheuristic. Each firefly in the population represents one potential solution to a problem and the population of fireflies should initially be distributed randomly and uniformly throughout the solution space. All FA solution procedures employ three specific rules: (i) The fireflies within a population are unisex, so that one firefly will be attracted to other fireflies irrespective of their sex; (ii) Attractiveness between any two fireflies is proportional to their brightness, implying that the less bright firefly will move towards the brighter one; and (iii) The explicit brightness of any firefly is explicitly determined by the corresponding value of its objective function. For maximization problems, the brightness can be considered proportional to the value of the objective function. Based upon these three rules, the basic operational steps of the FA can be summarized within the pseudo-code of Figure **1** [52].

```
Objective Function F(X), X = (x1, x2,... xd)
    Generate the initial population of n fireflies, Xi, i = 1, 2,..., n
    Light intensity Ii at Xi is determined by F(Xi)
    Define the light absorption coefficient γ
    while (t < MaxGeneration)
        for i = 1: n , all n fireflies
            for j = 1: n ,all n fireflies (inner loop)
                if (Ii<Ij), Move firefly i towards j; end if
                Vary attractiveness with distance r via e^-γr
            endfor j
        end for i
        Rank the fireflies and find the current global best solution G*
    end while
    Postprocess the results
```

Figure 1: Pseudo Code of the Firefly Algorithm.

There are two important requirements that must be determined for the FA: (i) the variation of light intensity and (ii) the formulation of attractiveness. Without loss of generality, it can always be assumed that the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function. In the simplest case, the brightness of a firefly at a particular location $\boldsymbol{X}$ would be its calculated objective value $F(\boldsymbol{X})$. However, the attractiveness, β, between fireflies is relative and will vary with the distance r_{ij} between firefly i and firefly j. In addition, light intensity decreases with the distance from its source, and light is also absorbed in the media, so the attractiveness needs to vary with the degree of absorption. Consequently, the overall attractiveness of a firefly can be defined as;

$$\beta = \beta_0 \exp(-\gamma r^2) \qquad \text{(A2)}$$

where β_0 is the attractiveness at distance $r = 0$ and γ is the fixed light absorption coefficient for the specific medium. If the distance r_{ij} between any two fireflies i and j located at $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$, respectively, is calculated using the Euclidean norm, then the movement of a firefly i that is attracted to another more attractive (i.e. brighter) firefly j is determined by;

$$\boldsymbol{X}_i = \boldsymbol{X}_i + \beta_0 \exp(-\gamma(r_{ij})^2)(\boldsymbol{X}_i - \boldsymbol{X}_j) + \alpha\boldsymbol{\varepsilon}_i \,. \qquad \text{(A3)}$$

In this expression of movement, the second term is due to the relative attraction and the third term is a randomization component. Yang [52] indicates that α is a randomization parameter normally selected within the range [0,1] and $\boldsymbol{\varepsilon}_I$ is a vector of random numbers drawn from either a Gaussian or uniform (generally [-0.5, 0.5]) distribution. It should be explicitly noted that this expression represents a random walk biased toward brighter fireflies and if $\beta_0 = 0$, it becomes a simple random walk. The parameter γcharacterizes the variation of the attractiveness and its value determines the speed of the algorithm's convergence. For most applications, γ is typically set between 0.1 to 10 [52, 58].

In any given optimization problem, for a very large number of fireflies $n >> k$, where k is the number of local optima, the initial locations of the n fireflies should be distributed relatively uniformly throughout the entire search space. As the FA proceeds, the fireflies begin to converge into all of the local optima (including the global ones). Hence, by comparing the best solutions among all these optima, the global optima can easily be determined. Yang [51] proves that the FA will approach the global optima when $n \rightarrow \infty$ and the number of iterations t, is set so that $t >>1$. In reality, the FA has been found to converge extremely quickly with n set in the range 20 to 50 [51, 58].

Two important limiting or asymptotic cases occur when $\gamma \rightarrow 0$ and when $\gamma \rightarrow \infty$. For $\gamma \rightarrow 0$, the attractiveness is constant $\beta = \beta_0$, which is equivalent to having a light intensity that does not decrease. Thus, a firefly would be visible to every other firefly anywhere within the solution domain. Hence, a single (usually global) optima can easily be reached. If the inner loop for j in Figure **1** is removed and $\boldsymbol{X}_j$ is replaced by the current global best $\boldsymbol{G^*}$, then this implies that the FA reverts to a special case of the accelerated particle swarm optimization (PSO) algorithm. Subsequently, the computational efficiency of this special FA case is equivalent to that of enhanced PSO. Conversely, when $\gamma \rightarrow \infty$, the attractiveness is essentially zero along the sightline of all other fireflies. This is equivalent to the case where the fireflies randomly roam throughout a very thick foggy region with no other fireflies visible and each firefly roams in a completely random fashion. This case corresponds to a completely random search method. As the FA operates between these two asymptotic extremes, it is possible to adjust the parameters α and γ so that the FA can outperform both a random search and the enhanced PSO algorithms [36].

The computational efficiencies of the FA will be exploited in the subsequent MGA solution approach. As noted, between the two asymptotic extremes, the population in the FA can determine both the global optima as well as the local optima concurrently. The concurrency of population-based solution procedures holds huge computational and efficiency advantages for MGA [38, 39]. An additional advantage of the FA for MGA implementation is that the different fireflies essentially work independently of each other, implying that FA procedures are better than genetic algorithms and PSO for MGA because the fireflies will tend to aggregate more closely around each local optimum [52, 58]. Consequently, with a judicious selection of parameter settings, the FA can be made to simultaneously converge extremely quickly into both local and global optima [51, 52, 58].

4. A SIMULATION-OPTIMIZATION APPROACH FOR STOCHASTIC OPTIMIZATION

The optimization of large stochastic problems proves to be very complicated when numerous system uncertainties have to be incorporated directly into the solution procedures [48-50, [57]. SO is a broadly defined family of stochastic solution approaches that combines simulation with an underlying optimization component for optimization [48]. In SO, all unknown objective functions, constraints, and parameters are replaced by discrete event simulation models in which the decision variables provide the settings under which the simulation is performed. While SO holds considerable potential for solving a wide range of difficult stochastic problems, it cannot be considered a panacea because of its accompanying processing time requirements [48, 49].

The general process of SO can be summarized in the following way [49, 57]. Suppose the mathematical representation of the optimization problem possesses n decision variables, X_i, expressed in vector format as $X = [X_1, X_2, \ldots, X_n]$. If the problem's objective function is designated by *F* and its feasible region is represented by *D*, then the related mathematical programming problem is to optimize *F*(***X***) subject to ***X*** ∊ *D*. When stochastic conditions exist, values for the constraints and objective are determined by simulation. Thus, any direct solution evaluation between two distinct solutions ***X1*** and ***X2*** requires the comparison of some statistic of *F* modelled with ***X1*** to the same statistic modelled with ***X2*** [20, 48]. These statistics are calculated by a simulation performed on the solutions, in which each candidate solution provides the decision variable settings in the simulation. While simulation presents a mechanism for comparing results, it does not provide the means for determining optimal solutions to problems. Hence, simulation, by itself, cannot be used as a stochastic optimization procedure.

Since all measures of system performance in SO are stochastic, every potential solution, ***X***, must be determined through simulation. Because simulation is computationally intensive, an optimization algorithm is employed to guide the search for solutions through the problem's feasible domain in as few simulation runs as possible [20, 50]. As stochastic system problems frequently contain numerous potential solutions, the quality of the final solution could be highly variable unless an extensive search has been performed throughout the problem's entire feasible region. Population-based metaheuristic such as the FA are conducive to these extensive searches because the complete set of candidate solutions maintained in their populations permit searches to be undertaken throughout multiple sections of the feasible region, concurrently.

An FA-directed SO approach contains two alternating computational phases; (i) an "evolutionary phase" directed by the FA module and (ii) a simulation module [5]. As described earlier, the FA maintains a population of candidate solutions throughout its execution. The evolutionary phase evaluates the entire current population of solutions during each generation of the search and evolves from the current population to a subsequent one. Because of the system's stochastic components, all performance measures are necessarily statistics calculated from the responses generated in the simulation module. The quality of each solution in the population is found by having its performance criterion, *F*, evaluated in the simulation module. After simulating each candidate solution, their respective objective values are returned to the evolutionary FA module to be utilized in the creation of the ensuing population of candidate solutions.

A primary characteristic of FA procedures is that better solutions in a current population possess a greater likelihood for survival and progression into the subsequent population. Thus, the FA module advances the system toward improved solutions in subsequent generations and ensures that the solution search does not become trapped in some local optima. After generating a new candidate population in the FA module, the new solution set is returned to the simulation module for comparative evaluation. This alternating, two-phase search process terminates when an appropriately stable system state (i.e. an optimal solution) has been attained. The optimal solution produced by the procedure is the single best solution found over the course of the entire search [5].

5. FA-DRIVEN SO ALGORITHM FOR STOCHASTIC MGA

Linton *et al*. [4] and Yeomans [20] have shown that SO can be used as a computationally intensive, stochastic MGA technique and these approaches have been applied to WRM problems [59-62]. Because of the very long computational runs, Yeomans [63] subsequently examined several approaches to accelerate the search times and solution quality of SO (see also [64]). This section parallels the framework of [57] in describing an FA-driven MGA method (see [5]) that incorporates stochastic uncertainty using SO to much more efficiently generate sets of maximally different solution alternatives.

The FA-driven stochastic MGA approach is designed to generate a pre-determined small number of close-to-optimal, but maximally different alternatives, by adjusting the value of *T* in [P1] and using the FA to solve each corresponding, maximal difference problem instance. This algorithm provides a stochastic extension to the deterministic approaches of [3, 55, 56]. By exploiting the co-evolutionary solution structure within the population of the FA, stratified subpopulations within the algorithm's overall population are established as the Fireflies collectively evolve toward different local optima within the solution space. In this process, each desired solution alternative undergoes the common search procedure driven by the FA. However, the survival of solutions depends not only upon how well the solutions perform with respect to the modelled objective(s), but also by how far away they are from all of the other alternatives generated in the decision space.

A direct process for generating these alternatives with the FA would be to iteratively solve the maximum difference model [P1] by incrementally updating the target *T* whenever a new alternative needs to be produced and then re-running the algorithm. Such an iterative approach would parallel the seminal Hop, Skip, and Jump (HSJ) MGA algorithm of [17] in which, once an initial problem formulation has been optimized, supplementary alternatives are created one-by-one through a systematic, incremental adjustment of the target constraint to force the sequential generation of the suboptimal solutions. While this direct approach is straightforward, it is relatively computationally expensive as it requires a repeated execution of the specific optimization algorithm employed [37-39, 53, 54].

In contrast, the concurrent FA-driven MGA approach is designed to generate the pre-determined number of maximally different alternatives within the entire population in a single run of the FA procedure (i.e. the same number of runs as if FA were used solely for function optimization purposes) and its efficiency is based upon the concept of co-evolution [53-56]. In this FA-driven co-evolutionary approach, pre-specified stratified subpopulation ranges within the FA's overall population are established that collectively evolve the search toward the creation of the stipulated number of maximally different alternatives. Each desired solution alternative is represented by each respective subpopulation and each subpopulation undergoes the common processing operations of the FA.

The FA-driven approach can be structured upon any standard FA solution procedure containing the appropriate encodings and operators that best correspond to the problem. The survival of solutions in each subpopulation depends simultaneously upon how well the solutions perform with respect to the modelled objective(s) and by how far away they are from all of the other alternatives. Consequently, the evolution of solutions in each subpopulation toward local optima is directly influenced by those solutions currently existing in all of the other subpopulations, which necessarily forces the concurrent co-evolution of each subpopulation towards good but maximally distant regions of the decision space. This co-evolutionary concept enables the simultaneous search for, and production of, the set of quantifiably good solutions that are maximally different from each other according to [P1] [39].

By employing this co-evolutionary concept, it becomes possible to implement an FA-driven MGA procedure that concurrently produces alternatives, which possess objective function bounds that are analogous, but inherently superior, to those created by a sequential HSJ-styled solution generation approach. While each alternative produced by an HSJ procedure is maximally different only from the single, overall optimal solution together with a bound on the objective value which is at least x% different from the best objective (i.e. x = 1%, 2%, etc.), the concurrent co-evolutionary FA procedure is able to generate alternatives that are no more than x% different from the overall optimal solution but with each one of these solutions being as maximally different as possible from every other generated alternative that is produced. Co-evolution is also much more efficient than a sequential HSJ-styled approach in that it exploits the inherent population-based searches of FA procedures to concurrently generate the entire set of maximally different solutions using only a single population. Specifically, while an HSJ-styled approach would need to run *n* different times in order to generate *n* different alternatives, the concurrent algorithm need run only once to produce its entire set of maximally different alternatives irrespective of the value of *n*. Hence, it is a much more computationally efficient solution generation process.

The steps involved in the stochastic FA-driven co-evolutionary MGA algorithm are as follows (see [57]):

(1) Create the initial population stratified into P equally-sized subpopulations. P represents the desired number of maximally different alternative solutions within a prescribed target deviation from the optimal to be generated and must be set *a priori* by the decision-maker. S_p represents the p^{th} subpopulation set of solutions, $p = 1,\ldots, P$ and there are K solutions contained within each S_p. Note that the target for each S_p could be a common deviation value (e.g. all P alternatives need to be within 10% of optimal) or the targets for each S_p could represent different selected increments (e.g. one alternative would need to be within 1% of optimal, another alternative would need to be within 2%, etc.).

(2) Evaluate each solution in S_1 using the simulation module and identify the best solution with respect to the modelled objective. S_1 is the subpopulation dedicated to the search for the overall optimal solution to the modelled problem. The best solution residing in S_1 is employed in establishing the benchmarks for the relaxation

constraints used to create the maximally different solutions as in P1.

(3) Evaluate all solutions in S_p, p = 2,…,P, with respect to the modelled objective using the simulation module. Solutions meeting the target constraint and all other problem constraints are designated as *feasible*, while all other solutions are designated as *infeasible*.

(4) Apply an appropriate elitism operator to each S_p to preserve the best individual in each subpopulation. In S_1, this is the best solution evaluated with respect to the modelled objective. In S_p, p = 2,…, P, the best solution is the feasible solution most distant in decision space from all of the other subpopulations (the distance measure is defined in Step 7). Note: Because the best solution to date is always placed into each subpopulation, at least one solution in S_p will always be feasible. This step simultaneously selects a set of alternatives that respectively satisfy different values of the target T while being as far apart as possible (i.e. maximally different in the sense of [P1]) from the solutions generated in each of the other subpopulations. By the co-evolutionary nature of this algorithm, the alternatives are simultaneously generated in one pass of the procedure rather than the P implementations suggested by the necessary HSJ-styled increments to T in problem [P1].

(5) Stop the algorithm if the termination criteria (such as maximum number of iterations or some measure of solution convergence) are met. Otherwise, proceed to Step 6.

(6) Identify the decision space centroid, C_{ip}, for each of the $K' \leq K$ feasible solutions within k = 1,…, K of S_p, for each of the N decision variables X_{ikp}, i = 1,…, N. Each centroid represents the N-dimensional centre of mass for the solutions in each of the respective subpopulations, p. As an illustrative example for determining a centroid, calculate $C_{ip} = (1/K') * \sum_k X_{ikp}$. In this calculation, each dimension of each centroid is computed as the straightforward average value of that decision variable over all of the values for that variable within the feasible solutions of the respective subpopulation. Alternatively, a centroid could be calculated as some fitness-weighted average or by some other appropriately defined measure.

(7) For each solution k = 1,…, K, in each S_q, calculate D_{kq}, a distance measure between that solution and all other subpopulations. As an illustrative example for determining a distance measure, calculate $D_{kq} = \text{Min}\ \{\sum_i |X_{ikp} - C_{ip}|;\ p = 1,\ldots, P,\ p \neq q\}$. This distance represents the minimum distance between solution k in subpopulation q and the centroids of all other subpopulations. Alternatively, the distance measure could be calculated by some other appropriately defined function.

(8) Rank the solutions within each S_p according to the distance measure D_{kq} objective – appropriately adjusted to incorporate any constraint violation penalties. The goal of maximal difference is to force solutions from one subpopulation to be as far apart as possible in the decision space from the solutions of each of the other subpopulations. This step orders the specific solutions in each subpopulation by those solutions which are most distant from the solutions in all of the other subpopulations.

(9) In each S_p, apply the appropriate FA “change operations” to the solutions and return to Step 2.

6. CASE STUDY OF WATER RESOURCES MANAGEMENT UNDER UNCERTAINTY

As indicated throughout the previous sections, decision-makers faced with situations containing numerous uncertainties generally prefer to be able to select from a set of “near best” alternatives that differ significantly from each other in terms of the system structures characterized by their decision variables. The effectiveness of the FA-driven SO MGA procedure will be illustrated using the water resources management case taken from [1] and [2]. While this section briefly outlines the case, more extensive details, data, and descriptions can be found in [1, 2, 60-62, 64].

[1] and [2] examined a water resources management case study for allocating water in a dry season from an unregulated reservoir to three categories of users: (i) a municipality, (ii) an industrial concern, and (iii) an agricultural sector. The industrial concern and agricultural sector were undergoing significant expansion and needed to know the quantities of water they could reasonably expect. If insufficient water were available, these entities would be forced to curtail their expansion plans. If the promised water was delivered, it would contribute positive net benefits to the local

economy per unit of water allocated. However, if the water was not delivered, the results would reduce the net benefits to the users.

The major problems in these circumstances involved (i) how to effectively allocate water to the three user groups in order to achieve maximum net benefits under the uncertain conditions and (ii) how to incorporate the water policies in terms of allowable amounts within this planning problem with the least risk of system disruption. Included within these decisions is a determination of which one of the multiple possible pathways that the water would flow through in reaching the users. It is further possible to subdivide the various water streams with each resulting sub stream sent to a different user. Since cost differences from operating the facilities at different capacity levels produce economies of scale, decisions have to be made to determine how much water should be sent along each flow pathway to each user type. Therefore, any single policy option can be composed of a combination of many decisions regarding which facilities received water and what quantities of water would be sent to each user type. All of these decisions were compounded by overriding system uncertainties regarding the seasonal water flows and their likelihoods.

Thus, the WRM case considers how to effectively allocate the water to the three user groups in order to derive maximum net benefits under the elements of uncertainty present and how to incorporate water policies in terms of allowable amounts within this planning problem with the least risk for causing system disruption. Since the uncertainties could be expressed collectively as interval estimates, probability distributions and uncertainty membership functions, the approach of [2] was used to show how to improve upon the earlier efforts of [1] by providing a solution for the WRM problem with a net benefit of $2.02 million.

6.1. Mathematical Model for The WRM Planning Case

This section briefly describes the stochastic programming method that [2] formulated to solve the WRM planning case. In the formulation, penalties are imposed when policies that have been expressed as targets are violated. Also within the model, any uncertain parameter *A* is represented by $A^{\pm}$ and its corresponding values are generated via probability distributions. More extensive details and descriptions of the model, and all of the underlying data for the parameter values, can be found in [1] and [2].

In the region studied, the municipal, industrial, and agricultural water demands have been increasing due to population and economic growth. Because of this, it is necessary to ensure that the different water users know where they stand by providing information that is needed to make decisions for various activities and investments. For example, farmers who know there is only a small chance of receiving sufficient water in a dry season are not likely to make major investment in irrigation infrastructure. Similarly, industries are not likely to promote developments of projects that are water intensive knowing that they will have to limit their water consumption. If the promised water cannot be delivered due to insufficiency, the users will have to either obtain water from more expensive alternate sources or curtail their development plans. For example, municipal residents may have to curtail watering of lawns, industries may have to reduce production levels or increase water recycling rates, and farmers may not be able to conduct irrigation as planned. These impacts will result in increased costs or decreased benefits in relation to the regional development. It is thus desired that the available water be effectively allocated to minimize any associated penalties. Thus, the problem can be formulated as maximizing the expected value of the net system benefits. Based upon the local water management policies, a quantity of water can be pre-defined for each user. If this quantity is delivered, it will result in net benefits; however, if not delivered, the system will then be subject to penalties.

The WRM authority is responsible for allocating water to each of the municipality, the industrial concerns, and the agricultural sector. As the quantity of stream flows from the reservoir are uncertain, the problem is formulated as a stochastic programming problem. This stochastic programming model can account for the uncertainties in water availability. However, uncertainties may also exist in other parameters such as benefits, costs and water-allocation targets. To reflect all of these uncertainties, the following stochastic programming model was constructed by [2]:

$$Max\, f^{\pm} = \sum_{i=1}^{m} B_i^{\pm} W_i^{\pm} - \sum_{i=1}^{m} \sum_{j=1}^{n} p_j C_i^{\pm} S_{ij}^{\pm} \tag{A4}$$

$$\sum_{i=1}^{m} \left(W_i^{\pm} - S_{ij}^{\pm} \right) \le q_j^{\pm} \qquad \forall\, j$$

$$S_{ij}^{\pm} \le W_i^{\pm} \le W_{i\max}^{\pm} \qquad \forall\ i$$

$$S_{ij}^{\pm} \ge 0 \qquad \forall\ i, j$$

In this formulation $f^{\pm}$ represents the net system benefit ($/m^3) and $B_i^{\pm}$ represents the net benefit to user *i* per m^3 of water allocated ($). $W_i^{\pm}$ is the fixed allocation amount (m^3) for water that is promised to user *i*, while $W_{i\max}^{\pm}$ is the maximum allowable amount (m^3) that can be allocated to user *i*. The loss to user *i* per m^3 of water not delivered is given by $C_i^{\pm}$, where C_i > B_i ($). $S_{ij}^{\pm}$ corresponds to the shortage of water, which is the amount (m^3) by which W_i is not met when the seasonal flow is q_j. $q_j^{\pm}$ is the amount (m^3) of seasonal flow with p_j probability of occurrence under *j* flow level, where p_j provides the probability (%) of occurrence of flow level *j*. The variable *i*, *i* = 1, 2, 3, designates the water user, where *i* = 1 for municipal, 2 for industrial, and 3 for agricultural. The value of *j* , *j* = 1, 2, 3, is used to delineate the flow level, where *j* = 1 represents low flows, 2 represents medium flows, and 3 represents high flows. Finally, *m* is the total number of water users and *n* is the total number of flow levels.

The developed formulation can provide results that are expressed as stable solutions with different risk levels within pre-established criteria [2]. This stochastic programming model holds two significant advantages in comparison to other optimization techniques that deal with uncertainties. Firstly, it enables the ability to reflect uncertainties expressed not only as probability distributions but also as possibility distributions. Secondly, it enables a linkage to be made with previously-existing or pre-defined policies that have to be respected whenever a modeling effort is undertaken. In this formulation, penalties are imposed when these policies, which are expressed as targets, are violated.

6.2. Using the Co-Evolutionary MGA Method for The WRM Planning Case

As outlined earlier, when public policy planners are faced with difficult and controversial choices, they generally prefer to be able to select from a set of near-optimal alternatives that differ significantly from each other in terms of their system structures. In order to create these alternative planning options for the WRM system, it would be possible to place extra target constraints into the original model which would force the generation of solutions that were different from their respective, initial optimal solutions. Suppose for example that five additional planning alternative options were created through the inclusion of a technical constraint on the objective function that decreased the total system benefits of the original model from 2% up to 10% in increments of 2%. By adding these incremental target constraints to the original SO model and sequentially resolving the problem 5 times, it would be possible to create a specific number of alternative policies for WRM planning.

However, to improve upon the process of running five separate additional instances of the computationally intensive SO algorithm to generate these solutions, the FA-driven MGA procedure described in the previous section was run only once, thereby producing the 5 additional alternatives shown in Table **1**. The table shows the overall system benefits for the 5 maximally different options generated. Given the performance bounds established for the objective in each problem instance, the decision-makers can feel reassured by the stated performance for each of these options while also being aware that the perspectives provided by the set of dissimilar decision variable structures are as different from each other as is feasibly possible. Hence, if there are stakeholders with incompatible standpoints holding diametrically opposing viewpoints, the policy-makers can perform an

Table 1: System Benefits ($ Millions) for 6 Maximally Different Alternatives

Maximally Different Solutions	WRM System Benefits ($ Millions)
Best Solution Overall	2.021
Best Solution Within 2%	1.987
Best Solution Within 4%	1.946
Best Solution Within 6%	1.915
Best Solution Within 8%	1.872
Best Solution Within 10%	1.840

assessment of these different options without being myopically constrained by a single overriding perspective based solely upon the objective value.

Furthermore, it should also be explicitly noted that the objective values for the alternatives created do not differ from the highest benefit solution by *at least* the stated 2%, 4%, …, 10%, respectively, but, in general, actually differ by less than these pre-specified upper deviation limits. This is because each of the best alternatives produced in S_2, S_3, S_4, S_5, S_6 have solutions whose structural variables differ maximally from those of all of the other alternatives generated while simultaneously guaranteeing that their objective values deviate from the overall best objective by *no more* than 2%, 4%,…, 10%, respectively. Thus, the goal of the alternatives generated in this MGA procedure are very different from those produced in the more straightforward HSJ-style, single-alternative-generation approach, while simultaneously establishing much more robust guarantees on the solution quality.

Although a mathematically optimal solution may not provide the best approach to the real problem, it can be demonstrated that the co-evolutionary procedure does indeed produce very good solution values to the originally modelled problem, itself. Table **1** clearly highlights how the alternative generated in S_1 by the MGA procedure is "good" with respect to the optimal solution found in [2]. In fact, it should be explicitly noted that the overall best solution produced by the MGA procedure (i.e. the solution in S_1) is actually identical to the one found by the function optimization approach of [2]. This is not mere coincidence because an expansion in the population size of the SO procedure to include the subpopulations S_2, S_3, …, S_6 does not detract from its evolutionary capabilities to find the best, function optimization solution in subpopulation S_1. Hence, in addition to its alternative generating capabilities, the MGA procedure simultaneously performs exceedingly well with respect to function optimization.

In summary, the computational example highlights several important features with respect to the FA-driven simulation-optimization MGA technique: (i) An FA can be effectively employed as the underlying optimization search routine for SO routines; (ii) Because of the evolving nature of its population-based solution searches, the co-evolutionary capabilities within the FA can be exploited to simultaneously generate more good alternatives than planners would be able to create using other MGA approaches; (iii) By the design of the MGA algorithm, the alternatives generated are good for planning purposes since all of their structures are guaranteed to be as mutually and maximally different from each other as possible (i.e. these differences are not just simply different from the overall optimal solution as in an HSJ-style approach to MGA); (iv) The approach is very computationally efficient since it need only be run once to generate its entire set of multiple, good solution alternatives (i.e. to generate *n* maximally different solution alternatives, the MGA algorithm would need to be run exactly the same number of times that the FA would need to be run for function optimization purposes alone – namely once – irrespective of the value of *n*); and, (v) The best overall solutions produced by the MGA procedure will be identical to the best overall solutions that would be produced by the FA for function optimization purposes alone.

CONCLUSIONS

WRM decision-making problems contain multi-faceted performance requirements which inevitably include complicated, incongruent performance objectives and unquantifiable modelling features. These problems often possess incompatible design specifications which are difficult – if not impossible – to capture when the supporting decision models are formulated. Consequently, there are unmodelled problem components, generally not apparent during model construction, that can significantly influence the acceptability of any model's solutions. These competing and ambiguous components force WRM decision-makers to incorporate many conflicting requirements into their decision process prior to settling upon a final solution.

Because of this, supplementary modelling techniques that support decision formulation must inherently capture the essence of these aspects while retaining sufficient flexibility to simultaneously consider the impacts from the planning and stochastic uncertainties. Rather than constructing exactly one, mathematically optimal solution, in these situations, it is more desirable to be able to generate a set of provably good options that provide distinctive perspectives to any potentially unmodelled issues. The distinctive structures captured by these dissimilar alternatives reflect very different system features, thereby addressing some of the unmodelled issues during the policy formulation stage.

This study has provided a stochastic FA-driven MGA approach that demonstrated how the co-evolutionary features of the FA could be employed to direct a stochastic SO search process to concurrently generate a set of maximally different, near-optimal alternatives. This stochastic MGA method creates several solutions containing the requisite problem features, with each alternative generated providing a very different perspective to the problem considered. The practicality of this FA-driven stochastic MGA approach can clearly be extended into numerous disparate environmental applications and can be readily modified to many other "real world" planning situations. Such extensions will be examined in forthcoming research initiatives.

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